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Supersymmetric intersecting branes on the type-IIA $T^6/\mathbb{Z}_4$ orientifold

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Abstract: We study supersymmetric intersecting D6-branes wrapping 3-cycles in the type-IIA $T^6/\mathbb{Z}_4$ orientifold background. As a new feature, the 3-cycles in this orbifold space arise both from the untwisted and the $\mathbb{Z}_2$ twisted sectors. We present an integral basis for the homology lattice, $H_3(M,\mathbb{Z})$, in terms of fractional 3-cycles, for which the intersection form involves the Cartan matrix of E_8. We show that these fractional D6-branes can be used to construct supersymmetric brane configurations realizing a three generation Pati-Salam model. Via brane recombination processes preserving supersymmetry, this GUT model can be broken down to a standard-like model.

Keywords: D-branes, Superstring Vacua, Conformal Field Models in String Theory.

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Contents

1. Introduction

Intersecting brane world models have been the subject of elaborate string model building for several years [1]–[38]. The main new ingredient in these models is that they contain intersecting D-branes and open strings in a consistent manner providing simple mechanisms to generate chiral fermions and to break supersymmetry [39, 40]. Most attempts

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for constructing realistic models were dealing with non-supersymmetric configurations of D-branes, mainly because non-trivial, chiral intersecting brane world models are not easy to find. It is known for instance that flat factorizing D-branes on the six-dimensional torus as well as on the $T^6/\mathbb{Z}_3$ orbifold can never give rise to globally supersymmetric models except for the trivial non-chiral configuration where all D6-branes are located on top of the orientifold plane [11]. Supersymmetric models clearly have some advantages over the non-supersymmetric ones. From the stringy point of view such models are stable, as not only the Ramond-Ramond (R-R) tadpoles cancel but also the Neveu-Schwarz-Neveu-Schwarz (NS-NS) tadpoles. From the phenomenological point of view, since the gauge hierarchy problem is solved by supersymmetry, one can work in the conventional scenarios with a large string scale close to the Planck scale or in an intermediate regime [41]. For an overview on other type-I constructions see [42].

The only semi-realistic supersymmetric models that have been found so far are defined in the $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold background and were studied in a series of papers [12, 13, 27, 28, 36]. Besides their phenomenological impact, type-IIA supersymmetric intersecting brane worlds with orientifold six-planes and D6-branes are also interesting from the stringy point of view, as they are expected to lift to M-theory on singular G_2 manifolds [43].

The aim of this paper is to pursue the study of intersecting brane worlds on orientifolds with a particular emphasis on the systematic construction of semi-realistic globally supersymmetric configurations. Note, that without the orientifold projection supersymmetric intersecting brane configurations do not exist, as the overall tension always would be positive. Interestingly, from the technical point of view, the $\mathbb{Z}_4$ orbifold involves some new insights, as not all 3-cycles are inherited from the torus. In fact, a couple of 3-cycles arise in the $\mathbb{Z}_2$ twisted sector implying that this model contains so-called fractional D6-branes, which have been absent in the $\mathbb{Z}_2 \times \mathbb{Z}_2$ and $\mathbb{Z}_3$ orbifolds. To treat these exceptional cycles accordingly, we will make extensive use of the formalism developed in [30].

It will turn out that supersymmetric models in general can be constructed in a straightforward way. But as in other model building approaches, finding semi-realistic three generation models turns out to be quite difficult. Fortunately, we will finally succeed in constructing a globally supersymmetric three generation Pati-Salam model with gauge group $\mathrm{SU}(4) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ and the Standard Model matter in addition to some exotic matter in the symmetric and antisymmetric representation of the two SU(2) gauge groups. In this paper, we will mainly focus on the new and interesting string model building aspects and leave a detailed investigation of the phenomenological implications of the discussed models for future work.

This paper is organized as follows. In section 2 we review some of the material presented in [30] about the general structure of intersecting brane worlds on Calabi-Yau manifolds. We will review those formulas which will be extensively used in the rest of the paper. In section 3 we start to investigate the $M = T^6/\mathbb{Z}_4$ orbifold and in particular derive an integral basis for the homology group $H_3(M, \mathbb{Z})$, for which the intersection form involves the Cartan-matrix of the Lie-algebra E_8. The main ingredient in the construction of such an integral basis will be the physical motivated introduction of fractional D-branes which also wrap around exceptional (twisted) 3-cycles in M. In section 4 we construct the

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orientifold models of type IIA on the orbifold M and discuss the orientifold planes, the action of the orientifold projection on the homology and the additional conditions arising for supersymmetric configurations. In section 5 we construct as a first example a globally supersymmetric four generation Pati-Salam model. Finally, in section 6 we elaborate on a supersymmetric model with initial gauge symmetry $U(4) \times U(2)^3 \times U(2)^3$ and argue that by brane recombination it becomes a supersymmetric three generation Pati-Salam model. By using conformal field theory methods, for this model we determine the chiral and also the massless non-chiral spectrum, which turns out to provide Higgs fields in just the right representations in order to break the model down to the Standard Model. At the end of the paper we describe both the GUT breaking and the electroweak breaking via brane recombination processes. We also make a prediction for the Weinberg angle at the string scale.

2. Intersecting Brane Worlds on Calabi-Yau spaces

Before we present our new model, we would like to briefly summarize some of the results presented in [30] about type-IIA orientifolds on smooth Calabi-Yau spaces. If the manifold admits an anti-holomorphic involution $\overline{\sigma}$, the combination $\Omega\overline{\sigma}$ is indeed a symmetry of the type-IIA model. Taking the quotient with respect to this symmetry introduces an orientifold six-plane into the background, which wraps a special lagrangian 3-cycle of the Calabi-Yau. In order to cancel the induced RR-charge, one introduces stacks of N_a D6-branes which are wrapped on 3-cycles π_a. Since under the action of $\overline{\sigma}$ such a 3-cycle, π_a, is in general mapped to a different 3-cycle, π'_a, one has to wrap the same number of D6-branes on the latter cycle, too. The equation of motion for the RR 7-form implies the RR-tadpole cancellation condition,

$$\sum_a N_a \left(\pi_a + \pi'_a\right) - 4\, \pi_{O6} = 0\,. \tag{2.1}$$

If it is possible to wrap a connected smooth D-brane on such an homology class, the stack of D6-branes supports a $U(N_a)$ gauge factor. Note, that it is not a trivial question if in a given homology class such a connected smooth manifold does exist. However, as we will see in section 6 for special cases, there are physical arguments ensuring that such smooth D-branes exist.

The Born-Infeld action provides an expression for the open string tree-level scalar potential which by differentiation leads to an equation for the NS-NS tadpoles

$$V = T_6\, \frac{e^{-\phi_4}}{M_s^3 \sqrt{\mathrm{Vol(M)}}} \left(\sum_a N_a \left(\mathrm{Vol}(\mathrm{D6}_a) + \mathrm{Vol}(\mathrm{D6}'_a)\right) - 4\, \mathrm{Vol(O6)} \right) \tag{2.2}$$

with the four-dimensional dilaton given by $e^{-\phi_4} = M_s^3 \sqrt{\mathrm{Vol(M)}} e^{-\phi_{10}}$ and T_6 denoting the tension of the D6-branes. By $\mathrm{Vol}(\mathrm{D6}_a)$ we mean the three dimensional internal volume of the D6-branes. Generically, this scalar potential is non-vanishing reflecting the fact that intersecting branes do break supersymmetry. If the cycles are special lagrangian (sLag) but calibrated with respect to 3-forms $\mathrm{Re}(e^{i\theta}\Omega_3)$ with different constant phase factors, the

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expression gets simplified to

$$V = T_6\, e^{-\phi_4} \left(\sum_a N_a \left| \int_{\pi_a} \widehat{\Omega}_3 \right| + \sum_a N_a \left| \int_{\pi'_a} \widehat{\Omega}_3 \right| - 4 \left| \int_{\pi_{\rm O6}} \widehat{\Omega}_3 \right| \right). \tag{2.3}$$

In this case, all D6-branes preserve some supersymmetry but not all of them the same. Models of this type have been discussed in [19, 21]. In the case of a globally supersymmetric model, all 3-cycles are calibrated with respect to the same 3-form as the O6-plane implying that the disc level scalar potential vanishes due to the RR-tadpole condition (2.1).

In [30] it was argued and confirmed by many examples that the chiral massless spectrum charged under the $\mathrm{U}(N_1) \times \ldots \times \mathrm{U}(N_k)$ gauge group of a configuration of k intersecting stacks of D6-branes can be computed from the topological intersection numbers as shown in table 1.

Representation	Multiplicity
$[\mathbf{A_a}]_L$	$\frac{1}{2}\left(\pi'_a \circ \pi_a + \pi_{\rm O6} \circ \pi_a\right)$
$[\mathbf{S_a}]_L$	$\frac{1}{2}\left(\pi'_a \circ \pi_a - \pi_{\rm O6} \circ \pi_a\right)$
$[(\overline{\mathbf{N}}_\mathbf{a}, \mathbf{N_b})]_L$	$\pi_a \circ \pi_b$
$[(\mathbf{N_a}, \mathbf{N_b})]_L$	$\pi'_a \circ \pi_b$

Table 1: Chiral spectrum in $d = 4$.

Since in six dimensions the intersection number between two 3-cycles is anti-symmetric, the self intersection numbers do vanish implying the absence of chiral fermions in the adjoint representation. Negative intersection numbers correspond to chiral fermions in the conjugate representations. Note, that if we want to apply these formulas to orientifolds on singular toroidal quotient spaces, the intersection numbers have to be computed in the orbifold space and not simply in the ambient toroidal space. After these preliminaries, we will discuss the $\mathbb{Z}_4$ orientifold in the following sections.

3. 3-cycles in the $\mathbb{Z}_4$ orbifold

We consider type-IIA string theory compactified on the orbifold background $T^6/\mathbb{Z}_4$, where the action of the $\mathbb{Z}_4$ symmetry, Θ, on the internal three complex coordinates reads

$$z_1 \to e^{\frac{\pi i}{2}}\, z_1\,, \qquad z_2 \to e^{\frac{\pi i}{2}}\, z_2\,, \qquad z_3 \to e^{-\pi i}\, z_3 \tag{3.1}$$

with $z_1 = x_1 + i x_2$, $z_2 = x_3 + i x_4$ and $z_3 = x_5 + i x_6$. This action preserves $\mathcal{N} = 2$ supersymmetry in four dimensions so that the orbifold describes a singular limit of a Calabi-Yau threefold. The Hodge numbers of this threefold are given by $h_{21} = 7$ and $h_{11} = 31$, where 1 complex and 5 Kähler moduli arise in the untwisted sector. The Θ and Θ^3 twisted sectors contain 16 $\mathbb{Z}_4$ fixed points giving rise to 16 additional Kähler moduli. In the Θ^2 twisted sector, there are 16 $\mathbb{Z}_2$ fixed points from which 4 are also $\mathbb{Z}_4$ fixed points. The latter ones contain 4 Kähler moduli whereas the remaining twelve $\mathbb{Z}_2$ fixed points are organized in pairs under the $\mathbb{Z}_4$ action giving rise to 6 complex and 6 Kähler moduli. The fact that the $\mathbb{Z}_2$ twisted sector contributes $h_{21}^{tw} = 6$ elements to the number of complex structure deformations and therefore contains what might be called twisted 3-cycles, is the salient new feature of this $\mathbb{Z}_4$ orbifold model as compared to the intersecting brane world models studied so far.

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Given this supersymmetric closed string background, we take the quotient by the orientifold projection $\Omega\overline{\sigma}$, where $\overline{\sigma}$ is an anti-holomorphic involution $z_i \to e^{i\phi_i}\overline{z}_i$ of the manifold. Note, that this orientifold model is not T-dual to the $\mathbb{Z}_4$ type-IIB orientifold model studied first in [44]. In the latter model there did not exist any supersymmetric brane configurations cancelling all tadpoles induced by the orientifold planes. In fact, as was pointed out in [45] our model is T-dual to a type-IIB orientifold on an asymmetric $\mathbb{Z}_4$ orbifold space. Slightly different $\mathbb{Z}_4$ type-IIB orientifold models were studied in [46, 47].

Our orientifold projection breaks supersymmetry in the bulk to $\mathcal{N}=1$ and introduces an orientifold $O6$-plane located at the fixed point locus of the anti-holomorphic involution. The question arises if one can introduce D6-branes, generically not aligned to the orientifold plane, in order to cancel the tadpoles induced by the presence of the $O6$ plane. The simplest such model where the D6-branes lie on top of the orientifold plane has been investigated in [48].

3.1 Crystallographic actions

Before dividing type-IIA string theory by the discrete symmetries $\mathbb{Z}_4$ and $\Omega\overline{\sigma}$, we have to ensure that the torus T^6 does indeed allow crystallographic actions of these symmetries. For simplicity, we assume that T^6 factorizes as $T^6 = T^2 \times T^2 \times T^2$. On the first two T^2s the $\mathbb{Z}_4$ symmetry enforces a rectangular torus with complex structure $U=1$. On each torus two different anti-holomorphic involutions

$$\mathbf{A}: \quad z_i \to \overline{z}_i \tag{3.2}$$

$$\mathbf{B}: \quad z_i \to e^{i\frac{\pi}{2}}\overline{z}_i \tag{3.3}$$

do exist. These two cases are shown in figure 1, where we have indicated the fixed point set of the orientifold projection $\Omega\overline{\sigma}$.[1]

Since on the third torus the $\mathbb{Z}_4$ acts like a reflection, the complex structure is unconstrained. But again there exist two different kinds of involutions, which equivalently correspond to the two possible choices of the orientation of the torus as shown in figure 2.

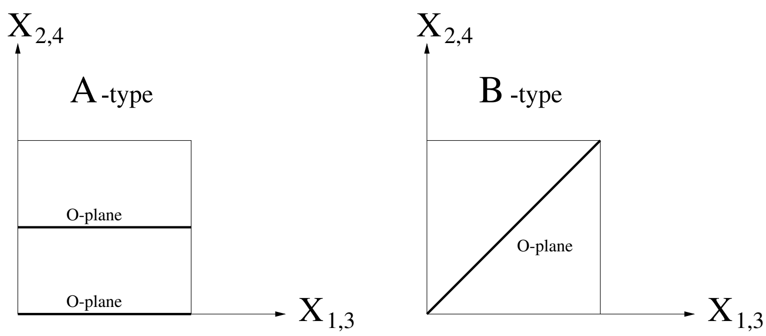

Figure 1: Anti-holomorphic involutions.

[1]The same distinction between the involutions **A** and **B** occurred for the first time in the papers [49, 50, 48, 51].

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For the **A**-torus the complex structure is given by $U = iU_2$ with U_2 unconstrained and for the **B**-torus the complex structure is given by $U = 1/2 + iU_2$. Therefore, by combining all possible choices of complex conjugations we get eight possible orientifold models. However, taking into account that the orientifold model on the $\mathbb{Z}_4$ orbifold does not only contain the orientifold planes related to $\Omega\overline{\sigma}$ but also the orientifold planes related to $\Omega\overline{\sigma}\Theta$, $\Omega\overline{\sigma}\Theta^2$ and $\Omega\overline{\sigma}\Theta^3$, only four models $\{$**AAA**,**ABA**,**AAB**,**ABB**$\}$ are actually different.

3.2 A non-integral basis of 3-cycles

In order to utilize the formulas from section 2, we have to find the independent 3-cycles on the $\mathbb{Z}_4$ orbifold space. Since we already know that the third Betti number, $b_3 = 2 + 2h_{21}$, is equal to sixteen, we expect to find precisely this number of independent 3-cycles.

One set of 3-cycles we get for free as they descend from the ambient space. Consider the three-cycles inherited from the torus T^6. We call the two fundamental cycles on the torus T^2_I $(I = 1, 2, 3)$ π_{2I-1} and π_{2I} and moreover we define the toroidal 3-cycles

$$\pi_{ijk} \equiv \pi_i \otimes \pi_j \otimes \pi_k \,. \tag{3.4}$$

Taking orbits under the $\mathbb{Z}_4$ action, one can deduce the following four $\mathbb{Z}_4$ invariant 3-cycles

$$\begin{aligned} \rho_1 &\equiv 2(\pi_{135} - \pi_{245})\,, \qquad & \bar{\rho}_1 &\equiv 2(\pi_{136} - \pi_{246}) \\ \rho_2 &\equiv 2(\pi_{145} + \pi_{235})\,, \qquad & \bar{\rho}_2 &\equiv 2(\pi_{146} + \pi_{236})\,. \end{aligned} \tag{3.5}$$

The factor of two in (3.5) is due to the fact that Θ^2 acts trivially on the toroidal 3-cycles. In order to compute the intersection form, we make use of the following fact: if the 3-cycles π^t_a on the torus are arranged in orbits of length N under some $\mathbb{Z}_N$ orbifold group, i.e.

$$\pi_a \equiv \sum_{i=0}^{N-1} \Theta^i \pi^t_a\,, \tag{3.6}$$

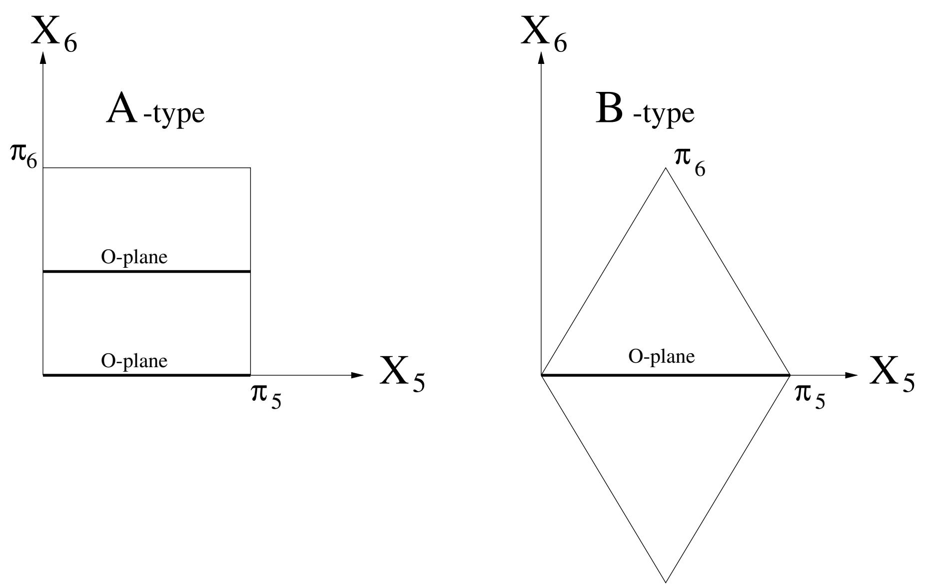

Figure 2: Orientations of the third T^2.

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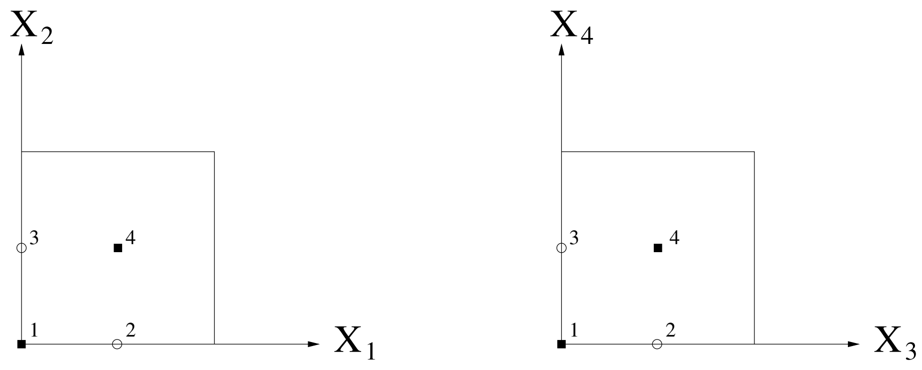

Figure 3: Orbifold fixed points.

the intersection number between two such 3-cycles on the orbifold space is given by

$$\pi_a \circ \pi_b = \frac{1}{N} \left(\sum_{i=0}^{N-1} \Theta^i \pi_a^t \right) \circ \left(\sum_{j=0}^{N-1} \Theta^j \pi_b^t \right) . \tag{3.7}$$

Therefore, the intersection form for the four 3-cycles (3.5) reads

$$I_\rho = \bigoplus_{i=1}^{2} \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} . \tag{3.8}$$

The remaining twelve 3-cycles arise in the $\mathbb{Z}_2$ twisted sector of the orbifold. Since Θ^2 acts non-trivially only onto the first two T^2, in the $\mathbb{Z}_2$ twisted sector the sixteen $\mathbb{Z}_2$ fixed points do appear as shown in figure 3.

The boxes in the figure indicate the $\mathbb{Z}_2$ fixed points which are also fixed under the $\mathbb{Z}_4$ symmetry. After blowing up the orbifold singularities, each of these fixed points gives rise an exceptional 2-cycle e_{ij} with the topology of S^2. These exceptional 2-cycles can be combined with the two fundamental 1-cycles on the third torus to form what might be called exceptional 3-cycles with the topology $S^2 \times S^1$. However, we have to take into account the $\mathbb{Z}_4$ action, which leaves four fixed points invariant and arranges the remaining twelve in six pairs. Since the $\mathbb{Z}_4$ acts by reflection on the third torus, its action on the exceptional cycles $e_{ij} \otimes \pi_{5,6}$ is

$$\Theta \left(e_{ij} \otimes \pi_{5,6} \right) = - e_{\theta(i)\theta(j)} \otimes \pi_{5,6} \tag{3.9}$$

with

$$\theta(1) = 1 \,, \qquad \theta(2) = 3 \,, \qquad \theta(3) = 2 \,, \qquad \theta(4) = 4 \,. \tag{3.10}$$

Due to the minus sign in (3.9) the invariant $\mathbb{Z}_4$ fixed points drop out and what remains are

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precisely the twelve 3-cycles

$$\begin{aligned}
\varepsilon_1 &\equiv (e_{12}-e_{13})\otimes\pi_5\,, &\quad \bar\varepsilon_1 &\equiv (e_{12}-e_{13})\otimes\pi_6\,,\\
\varepsilon_2 &\equiv (e_{42}-e_{43})\otimes\pi_5\,, &\quad \bar\varepsilon_2 &\equiv (e_{42}-e_{43})\otimes\pi_6\,,\\
\varepsilon_3 &\equiv (e_{21}-e_{31})\otimes\pi_5\,, &\quad \bar\varepsilon_3 &\equiv (e_{21}-e_{31})\otimes\pi_6\,,\\
\varepsilon_4 &\equiv (e_{24}-e_{34})\otimes\pi_5\,, &\quad \bar\varepsilon_4 &\equiv (e_{24}-e_{34})\otimes\pi_6\,,\\
\varepsilon_5 &\equiv (e_{22}-e_{33})\otimes\pi_5\,, &\quad \bar\varepsilon_5 &\equiv (e_{22}-e_{33})\otimes\pi_6\,,\\
\varepsilon_6 &\equiv (e_{23}-e_{32})\otimes\pi_5\,, &\quad \bar\varepsilon_6 &\equiv (e_{23}-e_{32})\otimes\pi_6\,.
\end{aligned} \tag{3.11}$$

Utilizing (3.7) the resulting intersection form is simply

$$I_\varepsilon = \bigoplus_{i=1}^{6}\begin{pmatrix} 0 & -2\\ 2 & 0\end{pmatrix}. \tag{3.12}$$

These 3-cycles lie in $H_3(M,\mathbb{Z})$ but do not form an integral basis of the free module since their intersection form is not unimodular.

3.3 An integral basis of 3-cycles

The cycles which are missing so far are the ones corresponding to what is called fractional D-branes [52, 53]. In our context these are D-branes wrapping only one-half times around the toroidal cycles $\{\rho_1, \bar\rho_1, \rho_2, \bar\rho_2\}$ while wrapping simultaneously around some of the exceptional 3-cycles. Therefore in the orbifold limit such branes are stuck at the fixed points and one needs at least two such fractional D-branes in order to form a brane which can be moved into the bulk.

To proceed, we need a rule of what combinations of toroidal and exceptional cycles are allowed for a fractional D-brane. Such a rule can be easily gained from our physical intuition. A D-brane wrapping for instance the toroidal cycle $\frac{1}{2}\rho_1$ can only wrap around those exceptional 3-cycles that correspond to the $\mathbb{Z}_2$ fixed points the flat D-brane is passing through. In our case, when the brane is lying along the x-axis on the three T^2s, the allowed exceptional cycles are $\{\varepsilon_1, \varepsilon_3, \varepsilon_5\}$. Therefore, the total homological cycle the D-brane is wrapping on can be for instance

$$\pi_a = \frac{1}{2}\rho_1 + \frac{1}{2}(\varepsilon_1 + \varepsilon_3 + \varepsilon_5)\,. \tag{3.13}$$

The relative signs for the four different terms in (3.13) are still free parameters and at the orbifold point do correspond to turning on a discrete Wilson line along a longitudinal internal direction of the D-brane. Note, that this construction is completely analogous to the construction of boundary states for fractional D-branes [54, 55, 56] carrying also a charge under some $\mathbb{Z}_2$ twisted sector states.

As an immediate consequences of this rule, only unbarred respectively barred cycles can be combined into fractional cycles, as they wrap the same fundamental 1-cycle on the third T^2. Apparently, the only non-vanishing intersection numbers are between barred and

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unbarred cycles. Any unbarred fractional D-brane can be expanded as

$$\pi_a = v_{a,1}\rho_1 + v_{a,2}\rho_2 + \sum_{i=1}^{6} v_{a,i+2}\,\varepsilon_i \tag{3.14}$$

with half-integer valued coefficients $v_{a,i}$. By exchanging the two fundamental cycles on the third T^2, we can associate to it a barred brane

$$\overline{\pi}_a = v_{a,1}\overline{\rho}_1 + v_{a,2}\overline{\rho}_2 + \sum_{i=1}^{6} v_{a,i+2}\,\overline{\varepsilon}_i \tag{3.15}$$

with the same coefficients $v_{a,i+8} = v_{a,i}$ for $i \in \{1,\ldots,8\}$. Using our rule we can construct all linear combinations with "self"-intersection number $\pi \circ \overline{\pi} = -2$, where we also have to keep in mind that the cycles form a lattice, i.e. integer linear combinations of cycles are again cycles.

In the following we list all the fractional 3-cycles with "self"-intersection number $\pi\circ\overline{\pi} = -2$. These cycles can be divided into 3 sets:

(*a*) $\{(v_1,v_2;v_3,v_4;v_5,v_6;v_7,v_8)|v_1+v_2 = \pm1/2,\ v_3+v_4 = \pm1/2,\ v_5+v_6 = \pm1/2,\ v_7+v_8 = \pm1/2;\ v_1+v_3+v_5+v_7 = 0 \bmod 1\}$. These combinations are obtained by observing which fixed points the flat branes parallel to the fundamental cycles do intersection These define $8\cdot 16 = 128$ different fractional 3-cycles.

(*b*) $\{(v_1,v_2;v_3,v_4;0,0;0,0),\ (v_1,v_2;0,0;v_5,v_6;0,0),\ (v_1,v_2;0,0;0,0;v_7,v_8),$ $(0,0;v_3,v_4;v_5,v_6;0,0),\ (0,0;v_3,v_4;0,0;v_7,v_8),\ (0,0;0,0;v_5,v_6;v_7,v_8)\ |v_i \in \pm1/2\}\}$. The first three kinds of cycles are again constructed from branes lying parallel to the x,y-axis on one T^2 and stretching along the diagonal on the other T^2. The remaining three kinds of cycles arise from integer linear combinations of the cycles introduced so far. Thus, in total this yields $6\cdot 16 = 96$ 3-cycles in the second set.

(*c*) $\{(v_1,v_2;v_3,v_4;v_5,v_6;v_7,v_8)|\text{exactly one } v_i = \pm1 \text{ ,rest zero}\}$. Only the vectors with $v_1 = \pm1$ or $v_2 \pm 1$ can be derived from untwisted branes. They are purely untwisted. The purely twisted ones again arise from linear combinations. This third set contains $2\cdot 8 = 16$ 3-cycles.

Altogether there are 240 of such 3-cycles with "self"-intersection number -2, which intriguingly just corresponds to the number of roots of the E_8 Lie algebra. Now, it is easy to write a computer program searching for a basis among these 240 cycles, so that the intersection form takes the following form

$$I = \begin{pmatrix} 0 & C_{E_8} \\ -C_{E_8} & 0 \end{pmatrix} \tag{3.16}$$

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where C_{E_8} denotes the Cartan matrix of E_8

$$C_{E_8} = \begin{pmatrix} -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -2 \end{pmatrix}. \tag{3.17}$$

One possible choice for the "simple roots" is

$$\begin{aligned} \vec{v}_1 &= \frac{1}{2}(-1,\ \ 0,-1,\ \ 0,-1,\ \ 0,-1,\ \ 0) \\ \vec{v}_2 &= \frac{1}{2}(\ \ 1,\ \ 0,\ \ 1,\ \ 0,\ \ 1,\ \ 0,-1,\ \ 0) \\ \vec{v}_3 &= \frac{1}{2}(\ \ 1,\ \ 0,-1,\ \ 0,-1,\ \ 0,\ \ 1,\ \ 0) \\ \vec{v}_4 &= \frac{1}{2}(-1,\ \ 0,\ \ 1,\ \ 0,\ \ 0,\ \ 1,\ \ 0,\ \ 1) \\ \vec{v}_5 &= \frac{1}{2}(\ \ 0,\ \ 1,-1,\ \ 0,\ \ 1,\ \ 0,\ \ 0,-1) \\ \vec{v}_6 &= \frac{1}{2}(\ \ 0,-1,\ \ 1,\ \ 0,-1,\ \ 0,\ \ 0,-1) \\ \vec{v}_7 &= \frac{1}{2}(\ \ 0,\ \ 1,\ \ 0,\ \ 1,\ \ 0,-1,\ \ 0,\ \ 1) \\ \vec{v}_8 &= \frac{1}{2}(\ \ 0,-1,\ \ 0,-1,\ \ 0,-1,\ \ 0,\ \ 1)\,. \end{aligned} \tag{3.18}$$

Since the Cartan matrix is unimodular, we indeed have constructed an integral basis for the homology lattice $H_3(M,\mathbb{Z})$. In the following, it turns out to be more convenient to work with the non-integral orbifold basis allowing also half-integer coefficients. However, as we have explained not all such cycles are part of $H_3(M,\mathbb{Z})$, so we have to ensure each time we use such fractional 3-cycles that they are indeed contained in the unimodular lattice $H_3(M,\mathbb{Z})$, i.e. that they are integer linear combinations of the basis (3.18).

4. Orientifolds on the $\mathbb{Z}_4$ orbifold

Equipped with the necessary information about the 3-cycles in the $\mathbb{Z}_4$ toroidal orbifold, we can move forward and consider the four inequivalent orientifold models in more detail.

4.1 The O6-planes

First, we have to determine the 3-cycle of the O6-planes. Let us discuss this computation for the **ABB** model in some more detail, as this orientifold will be of main interest for its potential to provide semi-realistic standard-like models.

We have to determine the fixed point sets of the four relevant orientifold projections $\{\Omega\overline{\sigma}, \Omega\overline{\sigma}\Theta, \Omega\overline{\sigma}\Theta^2, \Omega\overline{\sigma}\Theta^3\}$. The results are listed in table 2.

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Projection	fixed point set
$\Omega\,\overline{\sigma}$	$2\,\pi_{135}+2\,\pi_{145}$
$\Omega\,\overline{\sigma}\,\Theta$	$2\,\pi_{145}+2\,\pi_{245}-4\,\pi_{146}-4\,\pi_{246}$
$\Omega\,\overline{\sigma}\,\Theta^2$	$2\,\pi_{235}-2\,\pi_{245}$
$\Omega\,\overline{\sigma}\,\Theta^3$	$-2\,\pi_{135}+2\,\pi_{235}+4\,\pi_{136}-4\,\pi_{236}$

Table 2: O6-planes for **ABB** model.

Adding up all contributions we get

$$\begin{aligned} \pi_{O6} &= 4\,\pi_{145}+4\,\pi_{235}+4\,\pi_{136}-4\,\pi_{246}-4\,\pi_{146}-4\,\pi_{236} \\ &= 2\,\rho_2+2\,\overline{\rho}_1-2\,\overline{\rho}_2\,. \end{aligned} \tag{4.1}$$

Thus, only bulk cycles appear in π_{O6} reflecting the fact that in the conformal field theory the orientifold planes carry only charge under untwisted R-R fields [49, 48]. The next step is to determine the action of $\Omega\overline{\sigma}$ on the homological cycles. This can easily be done for the orbifold basis. We find for the toroidal 3-cycles

$$\begin{aligned} &\rho_1 \to \rho_2\,, \qquad \overline{\rho}_1 \to \rho_2-\overline{\rho}_2 \\ &\rho_2 \to \rho_1\,, \qquad \overline{\rho}_2 \to \rho_1-\overline{\rho}_1 \end{aligned} \tag{4.2}$$

and for the exceptional cycles

$$\begin{aligned} &\varepsilon_1 \to -\varepsilon_1 \qquad \overline{\varepsilon}_1 \to -\varepsilon_1+\overline{\varepsilon}_1 \\ &\varepsilon_2 \to -\varepsilon_2 \qquad \overline{\varepsilon}_2 \to -\varepsilon_2+\overline{\varepsilon}_2 \\ &\varepsilon_3 \to \varepsilon_3 \qquad \overline{\varepsilon}_3 \to \varepsilon_3-\overline{\varepsilon}_3 \\ &\varepsilon_4 \to \varepsilon_4 \qquad \overline{\varepsilon}_4 \to \varepsilon_4-\overline{\varepsilon}_4 \\ &\varepsilon_5 \to \varepsilon_6 \qquad \overline{\varepsilon}_5 \to \varepsilon_6-\overline{\varepsilon}_6 \\ &\varepsilon_6 \to \varepsilon_5 \qquad \overline{\varepsilon}_6 \to \varepsilon_5-\overline{\varepsilon}_5\,. \end{aligned} \tag{4.3}$$

Consistently, the orientifold plane (4.1) is invariant under the $\Omega\overline{\sigma}$ action. For the other three orientifold models, the results for the $O6$ planes and the action of $\Omega\overline{\sigma}$ on the homology lattice can be found in appendix A. In principle, we have now provided all the information that is necessary to build intersecting brane world models on the $\mathbb{Z}_4$ orientifold. However, since we are particularly interested in supersymmetric models we need to have control not only over topological data of the D6-branes but over the nature of the sLag cycles as well.

4.2 Supersymmetric cycles

The metric at the orbifold point is flat up to some isolated orbifold singularities. Therefore, flat D6-branes in a given homology class are definitely special lagrangian. We restrict our D6-branes to be flat and factorizable in the sense that they can be described by six wrapping numbers, (n_I, m_I) with $I = 1, 2, 3$, along the fundamental toroidal cycles, where for each I the integers (n_I, m_I) are relatively coprime. Given such a bulk brane, one can compute the homology class that it wraps expressed in the $\mathbb{Z}_4$ basis

$$\begin{aligned}\pi_a^{\rm bulk} &= [(n_{a,1}\, n_{a,2} - m_{a,1}\, m_{a,2}) n_{a,3}]\ \rho_1 + [(n_{a,1}\, m_{a,2} + m_{a,1}\, n_{a,2}) n_{a,3}]\ \rho_2 + \\ &\quad + [(n_{a,1}\, n_{a,2} - m_{a,1}\, m_{a,2}) m_{a,3}]\ \overline{\rho}_1 + [(n_{a,1}\, m_{a,2} + m_{a,1}\, n_{a,2}) m_{a,3}]\ \overline{\rho}_2\,. \end{aligned} \tag{4.4}$$

For the **ABB** orientifold, the condition that such a D6-brane preserves the same supersymmetry as the orientifold plane is simply

$$\varphi_{a,1} + \varphi_{a,2} + \varphi_{a,3} = \frac{\pi}{4} \bmod 2\pi \tag{4.5}$$

with

$$\tan\varphi_{a,1} = \frac{m_{a,1}}{n_{a,1}}\,, \qquad \tan\varphi_{a,2} = \frac{m_{a,2}}{n_{a,2}}\,, \qquad \tan\varphi_{a,3} = \frac{U_2\, m_{a,3}}{n_{a,3} + \frac{1}{2} m_{a,3}}\,. \tag{4.6}$$

Taking the $\tan(\ldots)$ on both sides of equation (4.5) we can reformulate the supersymmetry condition in terms of wrapping numbers (Note, that this only yields a necessary condition as $\tan(\ldots)$ is just periodic mod π.)

$$U_2 = \frac{\left(n_{a,3} + \frac{1}{2} m_{a,3}\right)}{m_{a,3}} \frac{(n_{a,1}\, n_{a,2} - m_{a,1}\, m_{a,2} - n_{a,1}\, m_{a,2} - m_{a,1}\, n_{a,2})}{(n_{a,1}\, n_{a,2} - m_{a,1}\, m_{a,2} + n_{a,1}\, m_{a,2} + m_{a,1}\, n_{a,2})}\,. \tag{4.7}$$

Therefore, the complex structure of the third torus in general is already fixed by one supersymmetric D-brane. In case one introduces more D6-branes, one gets non-trivial conditions on the wrapping numbers of these D-branes. The supersymmetry conditions for the other three orientifold models are summarized in appendix B.

Working only with the bulk branes (4.4), the model building possibilities are very restricted. In particular, it seems to be impossible to get large enough gauge groups to accommodate the Standard Model gauge symmetry, $\mathrm{U}(3)\times\mathrm{U}(2)\times\mathrm{U}(1)$, of at least rank six. One such supersymmetric model with only bulk branes and rank four has been constructed in [30]. Now, to enlarge the number of possibilities, we also allow such flat, factorizable branes to pass through $\mathbb{Z}_2$ fixed points and split into fractional D-branes. Thus, according to our rule we allow fractional D-branes wrapping the cycle

$$\pi_a^{\rm frac} = \frac{1}{2}\pi_a^{\rm bulk} + \frac{n_{a,3}}{2}\left[\sum_{j=1}^{6} w_{a,j}\,\varepsilon_j\right] + \frac{m_{a,3}}{2}\left[\sum_{j=1}^{6} w_{a,j}\overline{\varepsilon}_j\right] \tag{4.8}$$

with $w_{a,j} \in \{0, \pm 1\}$. To make contact with the formerly introduced coefficients $v_{a,j}$, we define

$$v_{a,j} = \frac{n_{a,3}}{2}\, w_{a,j}\,, \qquad v_{a,j+8} = \frac{m_{a,3}}{2}\, w_{a,j} \tag{4.9}$$

for $j \in \{1, \ldots, 8\}$. In (4.8) we have taken into account that the $\mathbb{Z}_2$ fixed points all lie on the first two two-dimensional tori and that on the third torus fractional D-branes do have winding numbers along the two fundamental 1-cycles. Moreover, since ε_j and $\overline{\varepsilon}_j$ only differ by the cycle on the third torus, their coefficients in (4.8) must indeed be equal.

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These fractional D6-branes do correspond to the following boundary states in the conformal field theory of the $T^6/\mathbb{Z}_4$ orbifold model

$$
\begin{aligned}
|D^f;(n_I,m_I),\alpha_{ij}\rangle \;=\; & \frac{1}{4\sqrt{2}}\left(\prod_{j=1}^{2}\sqrt{n_j^2+m_j^2}\right)\sqrt{n_3^2+n_3m_3+\frac{m_3^2}{2}}\Big(|D;(n_I,m_I)\rangle_U + \\
& \qquad\qquad + |D;\Theta(n_I,m_I)\rangle_U\Big) + \\
& +\frac{1}{2\sqrt{2}}\sqrt{n_3^2+n_3m_3+\frac{m_3^2}{2}}\left(\sum_{i,j=1}^{4}\alpha_{ij}|D;(n_I,m_I),e_{ij}\rangle_T + \right. \\
& \qquad\qquad \left. + \sum_{i,j=1}^{4}\alpha_{ij}|D;\Theta(n_I,m_I),\Theta(e_{ij})\rangle_T\right).
\end{aligned}
\tag{4.10}
$$

In the schematic form of the boundary state (4.10) there are contributions from both the untwisted and the $\mathbb{Z}_2$ twisted sector and we have taken the orbit under the $\mathbb{Z}_4$ symmetry Θ with the following action on the winding numbers

$$
\Theta(n_{1,2},m_{1,2}) = (-m_{1,2},n_{1,2}), \qquad \Theta(n_3,m_3) = -(n_3,m_3) \tag{4.11}
$$

implying that Θ^2 acts like the identity on the boundary states. This explains why only two and not four untwisted boundary states do appear in (4.10). Note, that in the sum over the $\mathbb{Z}_2$ fixed points, for each D6-brane precisely four coefficients take values $\alpha_{ij} \in \{-1,+1\}$ and the remaining ones are vanishing. The α_{ij} are of course directly related to the coefficients w_i appearing in the description of the corresponding fractional 3-cycles. For the interpretation of these coefficients α_{ij}, one has to remember that changing the sign of α_{ij} corresponds to turning on a discrete $\mathbb{Z}_2$ Wilson line along one internal direction of the brane [54, 56]. The action of Θ on the twisted sector ground states e_{ij} is the same as in (3.9). The elementary boundary states like $|D;(n_I,m_I)\rangle_U$ are the usual ones for flat D6 brane with wrapping numbers (n_I,m_I) on $T^6=T^2\times T^2\times T^2$ and can be found in appendix C. The important normalization factors in (4.10) are fixed by the Cardy condition, stating that the result for the annulus partition function must coincide for the loop and the tree channel computation.

Since the brane and its $\mathbb{Z}_4$ image only break the supersymmetry down to $\mathcal{N}=2$, one gets a $\mathcal{N}=2$ U(N) vector multiplet on each stack of fractional D-branes. The scalars in these vector multiplets correspond to the position of the D6-brane on the third T^2 torus, which is still an open string modulus.

Coming back to the homology cycles, following our general rule for fractional branes imposes further constraints on the coefficients because only those exceptional cycles are allowed to contribute which are intersected by the flat D-brane. The only allowed exceptional 3-cycles are summarized in table 3, depending on the wrapping numbers of the first two tori T^2.

At first glance, there is a mismatch between the number of parameters describing a 3-cycle and the corresponding boundary state. For each D6-brane there are three non-vanishing parameters w_i but four α_{ij}. However, a flat fractional brane and its $\mathbb{Z}_4$ image always intersect in precisely one $\mathbb{Z}_4$ fixed point times a circle on the third T^2.

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	n_1 odd, m_1 odd	n_1 odd, m_1 even	n_1 even, m_1 odd
n_2 odd m_2 odd		ε_3, ε_4 ε_5, ε_6	ε_3, ε_4 ε_5, ε_6
n_2 odd m_2 even	ε_1, ε_2 ε_5, ε_6	ε_1, ε_3, ε_5 ε_1, ε_4, ε_6 ε_2, ε_3, ε_6 ε_2, ε_4, ε_5	ε_1, ε_3, ε_6 ε_1, ε_4, ε_5 ε_2, ε_3, ε_5 ε_2, ε_4, ε_6
n_2 even m_2 odd	ε_1, ε_2 ε_5, ε_6	ε_1, ε_3, ε_6 ε_1, ε_4, ε_5 ε_2, ε_3, ε_5 ε_2, ε_4, ε_6	ε_1, ε_3, ε_5 ε_1, ε_4, ε_6 ε_2, ε_3, ε_6 ε_2, ε_4, ε_5

Table 3: Allowed exceptional cycles.

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Since Θ acts on this fixed locus with a minus sign, this twisted sector effectively drops out of the boundary state (4.10). A different way of saying this is that at the intersection between the brane and its $\mathbb{Z}_4$ image, there lives a hypermultiplet, Φ_{adj}, in the adjoint representation. Since it is an $\mathcal{N} = 2$ supermultiplet, there exists a flat direction in the D-term potential corresponding to the recombination of the two branes into a single brane. This single brane of course no longer runs to the $\mathbb{Z}_4$ invariant fixed point. This brane recombination process is depicted in figure 4.

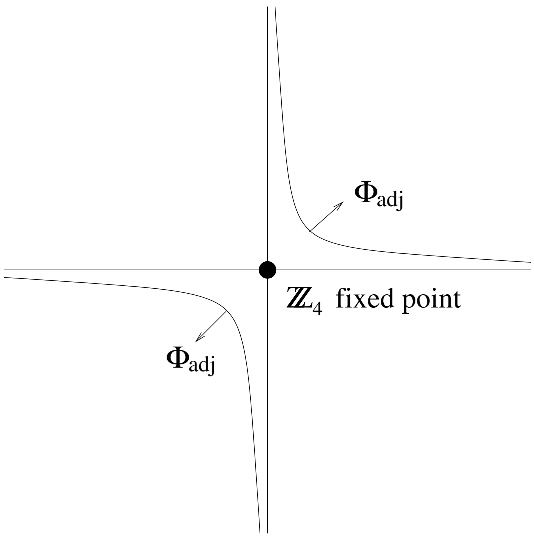

Figure 4: Recombined branes.

A non-trivial test for our considerations is the condition that a fractional brane (4.8) transformed to the E_8-basis must have integer coefficients. To see this, we write the 8×8 matrix (3.17) and a second identical copy as the two diagonal blocks of a 16×16 matrix, and then act with the inverse of the transposed matrix onto a general vector (4.8). Then we have to investigate the different cases according to table 3 separately. For instance for the case n_1 odd, n_2 odd, m_1 even, m_2 odd and fractional cycles ε_3, ε_4 with signs w_3, w_4 respectively, we substitute $m_1 = 2k_1$ and obtain the following vector in the E_8-basis:

$$\left[\Big(\frac{1}{2}(n_1 m_2 - w_3) + k_1 n_2\Big) n_3, \Big(\frac{1}{2}(n_1 n_2 - w_3) - k_1 m_2 + n_1 m_2 + 2k_1 n_2\Big) n_3, \ldots\right]. \qquad (4.12)$$

Already for the first two components we can see what generally happens for all cases and components: since n_1, n_2, m_2 and w_3 are non-vanishing and because products of odd numbers are also odd, just sums and differences of two odd numbers occur and these are always

stack	(n_I, m_I)	homology cycle
U(4) stack	$(-1,0;1,1;-1,0)$	$\pi_1 = \frac{1}{2}(\rho_1 + \rho_2 - \varepsilon_5 + \varepsilon_6)$ $\pi_1' = \frac{1}{2}(\rho_1 + \rho_2 + \varepsilon_5 - \varepsilon_6)$
U(2) stack	$(0,1;-1,-1;-1,2)$	$\pi_2 = \frac{1}{2}(-\rho_1 + \rho_2 + 2\overline{\rho}_1 - 2\overline{\rho}_2 + \varepsilon_5 - \varepsilon_6 - 2\overline{\varepsilon}_5 + 2\overline{\varepsilon}_6)$ $\pi_2' = \frac{1}{2}(-\rho_1 + \rho_2 + 2\overline{\rho}_1 - 2\overline{\rho}_2 + \varepsilon_5 - \varepsilon_6 - 2\overline{\varepsilon}_5 + 2\overline{\varepsilon}_6)$
U(2) stack	$(-1,0;1,-1;1,-2)$	$\pi_3 = \frac{1}{2}(-\rho_1 + \rho_2 + 2\overline{\rho}_1 - 2\overline{\rho}_2 - \varepsilon_5 + \varepsilon_6 + 2\overline{\varepsilon}_5 - 2\overline{\varepsilon}_6)$ $\pi_3' = \frac{1}{2}(-\rho_1 + \rho_2 + 2\overline{\rho}_1 - 2\overline{\rho}_2 - \varepsilon_5 + \varepsilon_6 + 2\overline{\varepsilon}_5 - 2\overline{\varepsilon}_6)$

Table 4: D6-branes for a 4 generation PS-model.

even or zero and therefore can be divided by 2 and still lead to integer coefficients. Having defined a well understood set of supersymmetric fractional D6-branes, we are now in the position to search for phenomenologically interesting supersymmetric intersecting brane worlds.

5. A four generation supersymmetric Pati-Salam model

In this section we present the construction of a semi-realistic supersymmetric intersecting brane world model. This provides an application of the formalism developed in the previous sections. It turns out that the **ABB** orientifold model is the most appropriate one for doing this. Using the fractional D6-branes introduced in the last section, one finds that by requiring that no (anti-)symmetric representations of the $\mathrm{U}(N_a)$ gauge groups do appear, only very few sufficiently small mutual intersection numbers arise. For the **ABB** model with the complex structure of the last torus being $U_2 = 1$, an extensive computer search reveals that essentially only mutual intersection numbers $(\pi_a \circ \pi_b, \pi_a' \circ \pi_b) = (0,0), (\pm 2, \mp 2)$ are possible. Even with these intersection numbers it is possible to construct a four generation supersymmetric Pati-Salam model with initial gauge group $\mathrm{U}(4) \times \mathrm{U}(2) \times \mathrm{U}(2)$. A typical model of this sort can be realized by the three stacks of D6-branes of table 4.

Computing the intersection numbers for these D6-branes and using the general formula for the chiral massless spectrum, one gets the massless modes shown in table 5.

n	$\mathrm{SU}(4) \times \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)^3$
2	$(4,2,1)_{(1,-1,0)}$
2	$(4,2,1)_{(1,1,0)}$
2	$(\overline{4},1,2)_{(-1,0,1)}$
2	$(\overline{4},1,2)_{(-1,0,-1)}$

Table 5: Chiral spectrum for 4 generation PS-model.

Here we have normalized as usual the gauge fields in the diagonal $\mathrm{U}(1)_a \subset \mathrm{U}(N_a)$ sub-algebras as

$$A^\mu_{\mathrm{U}(1)_a} = \frac{1}{N_a} \mathrm{Tr}\left(A^\mu_{\mathrm{U}(N_a)}\right). \tag{5.1}$$

Note, that all non-abelian gauge anomalies are canceled. Adding up all homological cycles, one finds that the RR-tadpole cancellation condition (2.1) is indeed satisfied. A nice check is whether the NS-NS tadpole cancellation condition (2.2) is also satisfied, as it should be for a globally supersymmetric configuration. For the contribution of the O6-plane to the

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scalar potential, one finds

$$\mathcal{V}_{O6} = -T_6\, e^{-\phi_4} 16\sqrt{2}\left(\frac{1}{\sqrt{U_2}} + 2\sqrt{U_2}\right), \tag{5.2}$$

whereas the three stacks of D6-branes give

$$\begin{aligned} \mathcal{V}_1 &= T_6\, e^{-\phi_4} 16\sqrt{2}\frac{1}{\sqrt{U_2}} \\ \mathcal{V}_{2,3} &= T_6\, e^{-\phi_4} 16\sqrt{2}\sqrt{U_2}\,. \end{aligned} \tag{5.3}$$

We see that the scalar potential vanishes for all values of the complex structure U_2 of the third torus. Thus, the disc level scalar potential indeed vanishes and we have constructed a globally supersymmetric intersecting brane world model with gauge group $\mathrm{U}(4) \times \mathrm{U}(2) \times \mathrm{U}(2)$.

5.1 Green-Schwarz mechanism

Computing in the usual way the mixed $\mathrm{U}(1)_a - \mathrm{SU}(N_b)^2$ anomalies, one confirms the general result derived in [30]

$$A_{ab} = \frac{N_a}{4}(-\pi_a + \pi'_a) \circ (\pi_b + \pi'_b)\,. \tag{5.4}$$

In our example there is only one anomalous U(1) while $\mathrm{U}(1)_2$ and $\mathrm{U}(1)_3$ are anomaly-free. This anomaly is canceled by some generalized Green-Schwarz mechanism involving the axionic couplings from the Chern-Simons terms in the effective action on the D6-branes

$$S^F_{CS} = \sum_{i=1}^{b_3} \int d^4x\, N_a\,(v_{a,i} - v'_{a,i})\, B_i \wedge F_a \tag{5.5}$$

and

$$S^{F\wedge F}_{CS} = \sum_{i=1}^{b_3} \int d^4x\,(v_{b,i} + v'_{b,i})\,\Phi_i\, \mathrm{Tr}(F_b \wedge F_b)\,, \tag{5.6}$$

where B_i is defined as the integral of the RR 5-form over the corresponding 3-cycle and similarly Φ_i is defined as the integral of the RR 3-form over the corresponding 3-cycle. Taking into account the Hodge duality between the fields B_i and Φ_{i+8} these axionic couplings indeed cancel the mixed anomalies. For more details we refer the reader to the general discussion in [30].

As was pointed out in [8] the couplings (5.5) can generate a mass term for U(1) gauge fields even if they are not anomalous. The massless U(1)s are given by the kernel of the matrix

$$M_{ai} = N_a(v_{a,i} - v'_{a,i})\,. \tag{5.7}$$

In our model it can be easily seen that $\mathrm{U}(1)_2$ and $\mathrm{U}(1)_3$ remain massless, so that the final gauge symmetry is $\mathrm{SU}(4) \times \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)^2$. We will not discuss this model any further but move forward to the construction of a more realistic model with three generations.

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6. A 3 generation supersymmetric Pati-Salam model

For the **ABB** model with the complex structure of the last torus fixed at $U_2 = 1/2$, a computer search shows that only sufficiently small mutual intersection numbers $(\pi_a \circ \pi_b, \pi'_a \circ \pi_b) = (0,0), (\pm 1, 0), (0, \pm 1)$ are possible. These numbers allow the construction of a three generation model in the following way. First, we start with seven stacks of D6-branes with an initial gauge symmetry $\mathrm{U}(4) \times \mathrm{U}(2)^6$ and choose the wrapping numbers as shown in table 6.

Adding up all homological 3-cycles, one realizes that the RR-tadpole cancellation condition is satisfied. The contribution of the O6-plane tension to the scalar potential is

$$V_{O6} = -T_6\, e^{-\phi_4} 16\sqrt{2} \left(\frac{1}{\sqrt{U_2}} + 2\sqrt{U_2} \right), \tag{6.1}$$

whereas the seven stacks of D6-branes give

$$\begin{aligned} V_1 &= T_6\, e^{-\phi_4} 16 \sqrt{\frac{1}{4\,U_2} + U_2} \\ V_{2,\ldots,7} &= T_6\, e^{-\phi_4} 8 \sqrt{\frac{1}{4\,U_2} + U_2}\,. \end{aligned} \tag{6.2}$$

Adding up all terms, one finds that indeed the NS-NS tadpole vanishes just for $U_2 = 1/2$. This means that in contrast to the four generation model, here supersymmetry really fixes the complex structure of the third torus. This freezing of moduli for supersymmetric backgrounds is very similar to what happens for instance in recently discussed compactifications with non-vanishing R-R fluxes [57, 58, 59].

stack	(n_I, m_I)	homology cycle
U(4)	$(1,0;1,0;0,1)$	$\pi_1 = \frac{1}{2}\left(\overline{\rho}_1 - \overline{\varepsilon}_1 - \overline{\varepsilon}_3 - \overline{\varepsilon}_5\right)$ $\pi'_1 = \frac{1}{2}\left(\rho_2 - \overline{\rho}_2 + \varepsilon_1 - \varepsilon_3 - \varepsilon_6 - \overline{\varepsilon}_1 + \overline{\varepsilon}_3 + \overline{\varepsilon}_6\right)$
U(2)	$(1,0;1,0;0,1)$	$\pi_2 = \frac{1}{2}\left(\overline{\rho}_1 - \overline{\varepsilon}_1 + \overline{\varepsilon}_3 + \overline{\varepsilon}_5\right)$ $\pi'_2 = \frac{1}{2}\left(\rho_2 - \overline{\rho}_2 + \varepsilon_1 + \varepsilon_3 + \varepsilon_6 - \overline{\varepsilon}_1 - \overline{\varepsilon}_3 - \overline{\varepsilon}_6\right)$
U(2)	$(1,0;1,0;0,1)$	$\pi_3 = \frac{1}{2}\left(\overline{\rho}_1 - \overline{\varepsilon}_2 + \overline{\varepsilon}_3 + \overline{\varepsilon}_6\right)$ $\pi'_3 = \frac{1}{2}\left(\rho_2 - \overline{\rho}_2 + \varepsilon_2 + \varepsilon_3 + \varepsilon_5 - \overline{\varepsilon}_2 - \overline{\varepsilon}_3 - \overline{\varepsilon}_5\right)$
U(2)	$(1,0;1,0;0,1)$	$\pi_4 = \frac{1}{2}\left(\overline{\rho}_1 + \overline{\varepsilon}_2 + \overline{\varepsilon}_3 + \overline{\varepsilon}_6\right)$ $\pi'_4 = \frac{1}{2}\left(\rho_2 - \overline{\rho}_2 - \varepsilon_2 + \varepsilon_3 + \varepsilon_5 + \overline{\varepsilon}_2 - \overline{\varepsilon}_3 - \overline{\varepsilon}_5\right)$
U(2)	$(1,0;1,0;0,1)$	$\pi_5 = \frac{1}{2}\left(\overline{\rho}_1 + \overline{\varepsilon}_1 - \overline{\varepsilon}_3 + \overline{\varepsilon}_5\right)$ $\pi'_5 = \frac{1}{2}\left(\rho_2 - \overline{\rho}_2 - \varepsilon_1 - \varepsilon_3 + \varepsilon_6 + \overline{\varepsilon}_1 + \overline{\varepsilon}_3 - \overline{\varepsilon}_6\right)$
U(2)	$(1,0;1,0;0,1)$	$\pi_6 = \frac{1}{2}\left(\overline{\rho}_1 + \overline{\varepsilon}_1 + \overline{\varepsilon}_4 - \overline{\varepsilon}_6\right)$ $\pi'_6 = \frac{1}{2}\left(\rho_2 - \overline{\rho}_2 - \varepsilon_1 + \varepsilon_4 - \varepsilon_5 + \overline{\varepsilon}_1 - \overline{\varepsilon}_3 + \overline{\varepsilon}_5\right)$
U(2)	$(1,0;1,0;0,1)$	$\pi_7 = \frac{1}{2}\left(\overline{\rho}_1 + \overline{\varepsilon}_1 - \overline{\varepsilon}_4 - \overline{\varepsilon}_6\right)$ $\pi'_7 = \frac{1}{2}\left(\rho_2 - \overline{\rho}_2 - \varepsilon_1 - \varepsilon_4 - \varepsilon_5 + \overline{\varepsilon}_1 + \overline{\varepsilon}_3 + \overline{\varepsilon}_5\right)$

Table 6: D6-branes for 3 generation PS-model.

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In terms of $\mathcal{N} = 2$ supermultiplets, the model contains vector multiplets in the gauge group $\mathrm{U}(4) \times \mathrm{U}(2)^3 \times \mathrm{U}(2)^3$ and in addition two hypermultiplets in the adjoint representation of each unitary gauge factor. The complex scalar in the vector multiplet corresponds to the unconstrained position of each stack of D6-branes on the third T^2. As described in section 4.2., the hypermultiplet appears on the intersection between a stack of branes and its $\mathbb{Z}_4$ image. By computing the intersection numbers, we derive the chiral spectrum as shown in table 7, where n denotes the number of chiral multiplets in the respective representation as given by the intersection number.

field	n	$\mathrm{U}(4) \times \mathrm{U}(2)^3 \times \mathrm{U}(2)^3$
$\Phi_{1'2}$	1	$(4; 2, 1, 1; 1, 1, 1)$
$\Phi_{1'3}$	1	$(4; 1, 2, 1; 1, 1, 1)$
$\Phi_{1'4}$	1	$(4; 1, 1, 2; 1, 1, 1)$
$\Phi_{1'5}$	1	$(\overline{4}; 1, 1, 1; \overline{2}, 1, 1)$
$\Phi_{1'6}$	1	$(\overline{4}; 1, 1, 1; 1, \overline{2}, 1)$
$\Phi_{1'7}$	1	$(\overline{4}; 1, 1, 1; 1, 1, \overline{2})$
$\Phi_{2'3}$	1	$(1; \overline{2}, \overline{2}, 1; 1, 1, 1)$
$\Phi_{2'4}$	1	$(1; \overline{2}, 1, \overline{2}; 1, 1, 1)$
$\Phi_{3'4}$	1	$(1; 1, \overline{2}, \overline{2}; 1, 1, 1)$
$\Phi_{5'6}$	1	$(1; 1, 1, 1; 2, 2, 1)$
$\Phi_{5'7}$	1	$(1; 1, 1, 1; 2, 1, 2)$
$\Phi_{6'7}$	1	$(1; 1, 1, 1; 1, 2, 2)$

Table 7: Chiral spectrum for a 7-stack model.

First, we notice that all non-abelian anomalies cancel including formally also the $\mathrm{U}(2)$ anomalies.

In order to proceed and really get a three generation model, it is necessary to break the two triplets $\mathrm{U}(2)^3$ down to their diagonal subgroups. Potential gauge symmetry breaking candidates in this way are the chiral fields $\{\Phi_{2'3}, \Phi_{2'4}, \Phi_{3'4}\}$ and $\{\Phi_{5'6}, \Phi_{5'7}, \Phi_{6'7}\}$ from table 7. However, one has to remember that these are chiral $\mathcal{N} = 1$ supermultiplets living on the intersection of two D-branes in every case. Let us review what massless bosons localized on intersecting D-branes indicate.

6.1 Brane recombination

If two stacks of D-branes preserve a common $\mathcal{N} = 2$ supersymmetry, then a massless hypermultiplet, H, localized on the intersection, signals a possible deformation of the two stacks of D-branes into recombined D-branes which wrap a complex cycle. Note, that two factorizable branes can only preserve $\mathcal{N} = 2$ supersymmetry if they are parallel on one of the three T^2_I tori. The complex cycle has the same volume as the sum of volumes of the two D-branes before the recombination process occurs. In the effective low energy theory, this recombination can be understood as a Higgs effect where a flat direction $\langle h_1 \rangle = \langle h_2 \rangle$ in the D-term potential

$$V_D = \frac{1}{2g^2} \left(h_1 \overline{h}_1 - h_2 \overline{h}_2 \right)^2 \tag{6.3}$$

exists, along which the $\mathrm{U}(N) \times \mathrm{U}(N)$ gauge symmetry is broken to the diagonal subgroup.[2] Here h_1 and h_2 denote the two complex bosons inside the hypermultiplet. Thus, in this case without changing the closed string background, there exists an open string modulus, which has the interpretation of a Higgs field in the low energy effective theory. Note, that in the T-dual picture, this is just the deformation of a small instanton into an instanton of finite

[2] If on one of the two stacks there sits only a single D6-brane, the F-term potential $\phi h_1 h_2$ forbids the existence of a flat direction with $\langle h_1 \rangle = \langle h_2 \rangle$. This is the field theoretic correspondence of the fact that there do not exist large instantons in the $\mathrm{U}(1)$ gauge group. We thank A. Uranga for pointing this out to us.

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size. In our concrete models such $\mathcal{N} = 2$ Higgs sectors are coupled at brane intersections to chiral $\mathcal{N} = 1$ sectors. Note, that the brane recombination in the effective gauge theory cannot simply be described by the renormalizable couplings. In order to get the correct light spectrum, one also has to take into account stringy higher dimensional couplings.

When the two D-branes only preserve $\mathcal{N} = 1$ supersymmetry and support a massless chiral supermultiplet Φ on the intersection [60, 61], the situation gets a little bit more involved. In this case, the analogous D-term potential is of the form

$$V_D = \frac{1}{2g^2} \left(\phi\overline{\phi}\right)^2 \tag{6.4}$$

which tells us that, unless there are more chiral fields involved, simply by giving a VEV to the massless boson ϕ, we do not obtain a flat direction of the D-term potential and therefore break supersymmetry. Nevertheless, the massless modes indicate that the intersecting brane configuration lies on a line of marginal stability in the complex structure moduli space. By a small variation of the complex structure, a Fayet-Iliopoulos (FI) term, r, is introduced that changes the D-term potential to

$$V_D = \frac{1}{2g^2} \left(\phi\overline{\phi} + r\right)^2 . \tag{6.5}$$

Therefore, for $r < 0$ the field ϕ becomes tachyonic and there exists a new stable supersymmetric minimum of the D-term potential. The intersecting branes then have combined into one D-brane wrapping a special lagrangian 3-cycle in the underlying Calabi-Yau. For a finite FI-term r, this 3-cycle has smaller volume than the two intersecting branes. However, the two volumes are precisely equal on the line of marginal stability. This means that on the line of marginal stability, there exists a different configuration with only a single brane which also preserves the same $\mathcal{N} = 1$ supersymmetry and has the same volume as the former pair of intersecting D-branes. Again the gauge symmetry is broken to the diagonal subgroup. It has to be emphasized that in this case the two configurations are not simply linked by a Higgs mechanism in the effective low energy gauge theory. As mentioned before, in order to deform the intersecting brane configuration into the non-flat D-brane wrapping a special lagrangian 3-cycle, one first has to deform the closed string background and then let the tachyonic mode condense. Therefore, the description of this process is intrinsically stringy and should be better described by string field theory rather than the effective low energy gauge theory.[3] For $r > 0$, the non-supersymmetric intersecting branes are stable and have a smaller volume than the recombined brane. The lift of these brane recombination processes to M-theory was discussed in [62].

[3]In the context of so-called quasi-supersymmetric intersecting brane world models [21], it has been observed that indeed the brane recombination of $\mathcal{N} = 1$ supersymmetric intersections cannot simply be described by a Higgs mechanism of massless modes. It was suggested there that the stringy nature of this transition has the meaning that also some massive, necessarily non-chiral, fields are condensing during the brane recombination. At least from the effective gauge theory point of view, this could induce the right mass terms which are necessary for an understanding of the new massless modes after the recombination. We leave it for future work to find the right effective description of this transition, but we can definitely state that it must involve some stringy aspects as the complex structure changes, i.e. the closed string background.

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After this little excursion, we come back to our model. We have seen that the condensation of hypermultiplets is under much better control than the condensation of chiral multiplets. Therefore, we have to determine the Higgs fields in our model as well, meaning to compute the non-chiral spectrum. This cannot be done by a simple homology computation, but fortunately we do know the exact conformal field theory at the orbifold point. Using the boundary states (4.10), we can determine the non-chiral matter living on intersections of the various stacks of D-branes. One first computes the overlap between two such boundary states and then transforms the result to the open string channel to get the annulus partition function, from which one can read off the massless states. This is a straightforward but tedious computation, which also confirms the chiral spectrum in table 7. Thus the conformal field theory result agrees completely with the purely topological computation of the intersection numbers.

Computing the non-chiral spectrum just for one stack of U(2) branes and their $\mathbb{Z}_4$ and $\Omega\overline{\sigma}$ images, one first finds the well known hypermultiplet, $\Phi_{adj} = (\phi_{adj}, \tilde{\phi}_{adj})$, in the adjoint representation of U(2) localized on the intersection of a brane and its $\mathbb{Z}_4$ image. Moreover, there are two chiral multiplets, Ψ_A and $\Psi_{\overline{A}}$, in the $\mathbf{A}$ respectively $\overline{\mathbf{A}}$ representation arising from the (π_i, π_i') sector. Since the two chiral fields carry conjugate representations of the gauge group, they cannot be seen by the topological intersection number which in fact vanishes, $\pi_i \circ \pi_i' = 0$. We have depicted the resulting quiver diagram for these three fields in figure 5.

For each closed polygon in the quiver diagram, the associated product of fields can occur in the holomorphic superpotential. In our case, the following two terms can appear

$$W = \phi_{adj}\, \Psi_A \Psi_{\overline{A}} + \tilde{\phi}_{adj}\, \Psi_A \Psi_{\overline{A}}\,, \tag{6.6}$$

which generate a mass for the anti-symmetric fields when the adjoint multiplet gets a VEV.

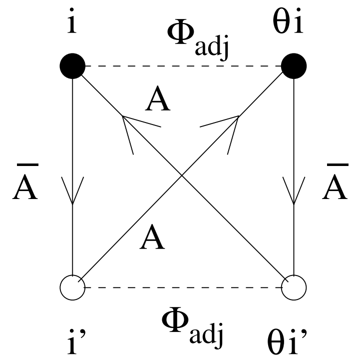

Figure 5: Adjoint higgsing.

As we have mentioned already in the last section, giving a VEV to this adjoint field localized on the intersection between a brane and its $\mathbb{Z}_4$ image, leads to the recombination of these two branes. The recombined brane no longer passes through the $\mathbb{Z}_4$ invariant intersection points.

After computing all annulus partition functions for pairs of D-branes from table 6, we find the total non-chiral spectrum listed in table 8.

It is interesting that we find Higgs fields which might break a SU(4) × SU(2) × SU(2) gauge symmetry in a first step down to the Standard Model and in a second step down to SU(3) × U(1)$_{em}$. However, the Higgs fields which would allow us to break the product groups U(2)^3 down to their diagonal subgroup are not present in the non-chiral spectrum.

6.2 D-flatness

However, we do have the massless chiral bifundamental fields $\{\Phi_{2'3}, \ldots, \Phi_{6'7}\}$ living on intersections preserving $\mathcal{N} = 1$ supersymmetry. As we have already mentioned, for isolated brane intersections these massless fields indicate that the complex structure moduli

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are chosen such that one sits on a line of marginal stability. On one side of this line, the intersecting branes break supersymmetry without developing a tachyonic mode. This indicates that the intersecting brane configuration is stable. But on the other side of the line, the former massless chiral field becomes tachyonic and after condensation leads to a new in general non-flat supersymmetric brane wrapping a special lagrangian 3-cycle. Since the tachyon transforms in the bifundamental representation, on this brane the gauge symmetry is broken to its diagonal subgroup.

We therefore expect for our compact situation that at least locally these bifundamental chiral multiplets indicate the existence of a recombined brane of the same volume but with the gauge group broken to the diagonal subgroup. In order to make our argument save, we need to show that the D-terms allow, that for certain continuous deformations of the complex structure moduli, just the four fields $\{\Phi_{2'3}, \Phi_{2'4}, \Phi_{5'6}, \Phi_{5'7}\}$ become tachyonic. Then they condense to a new supersymmetric ground state and the gauge symmetries $\mathrm{U}(2)^3$ are broken to the diagonal U(2)s. From general arguments for open string models with $\mathcal{N}=1$ supersymmetry, it is known that the complex structure moduli only appear in the D-term potential, whereas the Kähler moduli only appear in the F-term potential [63, 64, 65, 66].

field	n	$\mathrm{U}(4)\times \mathrm{U}(2)^3 \times \mathrm{U}(2)^3$
H_{12}	1	$(4;\overline{2},1,1;1,1,1)$ + c.c.
H_{13}	1	$(4;1,\overline{2},1;1,1,1)$ + c.c.
H_{14}	1	$(4;1,1,\overline{2};1,1,1)$ + c.c.
H_{15}	1	$(\overline{4};1,1,1;2,1,1)$ + c.c.
H_{16}	1	$(\overline{4};1,1,1;1,2,1)$ + c.c.
H_{17}	1	$(\overline{4};1,1,1;1,1,2)$ + c.c.
H_{25}	1	$(1;2,1,1;\overline{2},1,1)$ + c.c.
H_{26}	1	$(1;2,1,1;1,\overline{2},1)$ + c.c.
H_{27}	1	$(1;2,1,1;1,1,\overline{2})$ + c.c.
H_{35}	1	$(1;1,2,1;\overline{2},1,1)$ + c.c.
H_{36}	1	$(1;1,2,1;1,\overline{2},1)$ + c.c.
H_{37}	1	$(1;1,2,1;1,1,\overline{2})$ + c.c.
H_{45}	1	$(1;1,1,2;\overline{2},1,1)$ + c.c.
H_{46}	1	$(1;1,1,2;1,\overline{2},1)$ + c.c.
H_{47}	1	$(1;1,1,2;1,1,\overline{2})$ + c.c.

Table 8: Non-chiral spectrum (Higgs fields).

Remember that the Green-Schwarz mechanism requires the Chern-Simons couplings to be of the form

$$S_{CS} = \sum_{i=1}^{b_3} \sum_{a=1}^{k} \int d^4x\, M_{ai}\, B_i \wedge \frac{1}{N_a}\, \mathrm{tr}(F_a)\,. \tag{6.7}$$

The supersymmetric completion involves a coupling of the auxiliary field D_a

$$S_{FI} = \sum_{i=1}^{b_3} \sum_{a=1}^{k} \int d^4x\, M_{ai}\, \frac{\partial \mathcal{K}}{\partial \phi_i}\, \frac{1}{N_a}\, \mathrm{tr}(D_a)\,, \tag{6.8}$$

where ϕ_i are the superpartners of the Hodge duals of the RR 2-forms and $\mathcal{K}$ denotes the Kähler potential. Thus, these couplings give rise to FI-terms depending on the complex structure moduli which we parameterize simply by $A_i = \partial\mathcal{K}/\partial\phi_i$.[4]

Let us now discuss the D-term potential for the $\mathrm{U}(4)\times\mathrm{U}(2)^3\times\mathrm{U}(2)^3$ gauge fields in our model and see whether it allows supersymmetric ground states of the type described above. The D-term potential including only the chiral matter and the FI-terms in general

[4]For our purposes we do not need the precise form of the Kähler potential as long as the map from the complex structure moduli ϕ_i to the new parameters A_i is one to one. But this is the case, as the functional determinant for the map between these two sets of variables is equal to $\mathrm{Det}\left(\frac{\partial^2\mathcal{K}}{\partial\phi_i\partial\phi_j}\right)$, which is non-vanishing for a positive definite metric on the complex structure moduli space.

reads

$$V_D = \sum_{a=1}^{k} \sum_{r,s=1}^{N_a} \frac{1}{2g_a^2} (D_a^{rs})^2$$
$$= \sum_{a=1}^{k} \sum_{r,s=1}^{N_a} \frac{1}{2g_a^2} \left(\sum_{j=1}^{k} \sum_{p=1}^{N_j} q_{aj}\, \Phi_{aj}^{rp}\, \overline{\Phi}_{aj}^{sp} + g_a^2 \sum_{i=1}^{b_3} \frac{M_{ai}}{N_a}\, A_i\, \delta^{rs} \right)^2 , \qquad (6.9)$$

where the indices (r,s) numerate the N_a^2 gauge fields in the adjoint representation of the gauge factor $\mathrm{U}(N_a)$ and the sum over j is over all chiral fields charged under $\mathrm{U}(N_a)$. The gauge coupling constants depend on the complex structure moduli as well, but since we are only interested in the leading order effects, we can set them to the constant values on the line of marginal stability. Since all branes have the same volume there, in the following we will simply set them to one. In our case, the charges q_{aj} can be read off from table 7 and the Green-Schwarz couplings M_{ai} from table 6 using the definition (5.7). It is then straightforward to show that for the following non-vanishing $\Omega\overline{\sigma}$ invariant complex structure deformations related to the four 3-cycles $\{\varepsilon_1, \varepsilon_2, \varepsilon_3 - 2\overline{\varepsilon}_3, \varepsilon_4 - 2\overline{\varepsilon}_4\}$

$$A_3 = -\kappa\,, \qquad A_5 - 2A_{13} = -\kappa$$
$$A_4 = \kappa - 2\lambda\,, \qquad A_6 - 2A_{14} = 2\mu - \kappa \qquad (6.10)$$

just the fields $\{\Phi_{2'3}, \Phi_{2'4}, \Phi_{5'6}, \Phi_{5'7}\}$ become tachyonic. There exists a new supersymmetric ground state for the non-vanishing VEVs

$$|\Phi_{2'3}^{rr}|^2 = \lambda\,, \qquad |\Phi_{2'4}^{rr}|^2 = \kappa - \lambda$$
$$|\Phi_{5'6}^{rr}|^2 = \mu\,, \qquad |\Phi_{5'7}^{rr}|^2 = \kappa - \mu \qquad (6.11)$$

with $r = 1, 2$, $\lambda, \mu > 0$, $\kappa > \lambda$ and $\kappa > \mu$. From this small calculation, we conclude that our model indeed sits on a locus of marginal stability, for which a supersymmetric configuration exists where the branes $\{\pi_2, \pi_3', \pi_4'\}$ and similarly the branes $\{\pi_5, \pi_6', \pi_7'\}$ have recombined into a single stack of branes within the same homology class.

6.3 Gauge symmetry breaking

After this recombination process we are left with only three stacks of D6-branes wrapping the homology cycles

$$\pi_a = \pi_1, \quad \pi_b = \pi_2 + \pi_3' + \pi_4'\,, \qquad \pi_c = \pi_5 + \pi_6' + \pi_7'\,. \qquad (6.12)$$

These branes are not factorizable but we have presented arguments ensuring that they preserve the same supersymmetry as the closed string sector and the former intersecting brane configuration.[5] The chiral spectrum for this now 3 stack model is shown in table 9.

The intersection numbers $\pi_{b,c}' \circ \pi_{b,c}$ do not vanish any longer, therefore giving rise to chiral multiplets in the symmetric and anti-symmetric representation of the $\mathrm{U}(2)$ gauge

[5] Since we get chiral fields in the (anti-)symmetric representations after brane recombination, one might check if those intersection numbers can also be obtained by flat factorizable D-branes. Remember that we had the first assumption that there are no such chiral fields in the (anti-)symmetric representations. In fact, after an extensive computer search we have not been able to find a model with just factorizable D-branes generating the chiral spectrum of table 9.

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factors. Clearly, these chiral fields are needed in order to cancel the formal non-abelian U(2) anomalies. These anti-symmetric fields can be understood as the remnants of the chiral fields, $\Phi_{3'4}$ and $\Phi_{6'7}$, which did not condense during the brane recombination process.

Computing the mixed anomalies for this model, one finds that two U(1) gauge factors are anomalous and that the only anomaly free combination is

$$U(1) = U(1)_a - 3\, U(1)_b - 3\, U(1)_c\,. \tag{6.13}$$

field	n	$SU(4) \times SU(2) \times SU(2) \times U(1)^3$
Φ_{ab}	2	$(4, 2, 1)_{(1,-1,0)}$
$\Phi_{a'b}$	1	$(4, 2, 1)_{(1,1,0)}$
Φ_{ac}	2	$(\overline{4}, 1, 2)_{(-1,0,1)}$
$\Phi_{a'c}$	1	$(\overline{4}, 1, 2)_{(-1,0,-1)}$
$\Phi_{b'b}$	1	$(1, S + A, 1)_{(0,2,0)}$
$\Phi_{c'c}$	1	$(1, 1, \overline{S} + \overline{A})_{(0,0,-2)}$

Table 9: Chiral spectrum for 3 stack PS-model.

The quadratic axionic couplings reveal that the matrix M_{ai} in (5.7) has a trivial kernel and therefore all three U(1) gauge groups become massive and survive as global symmetries. To summarize, after the recombination of some of the $U2$) branes we have found a supersymmetric 3 generation Pati-Salam model with gauge group $SU(4) \times SU(2)_L \times SU(2)_R$ which accommodates the standard model matter in addition to some exotic matter in the (anti-)symmetric representation of the SU(2) gauge groups.

To compute the massless non-chiral spectrum after the recombination, we have to determine which Higgs fields receive a mass from couplings with the condensing chiral bifundamental fields. As we have explained earlier, the applicability of the low energy effective field theory is limited but still is the only information we have. So, we will see how far we can get. We first consider the sector of the branes $\{\pi_1, \ldots, \pi_4\}$ in figure 6.

The chiral fields are indicated by an arrow and non-chiral fields by a dashed line. The fields which receive a VEV after small complex structure deformations are depicted by a fat line. Let us decompose the Higgs fields inside one hypermultiplet into its two components $H_{1j} = (h^{(1)}_{1j}, h^{(2)}_{1j})$ for $j = 2, 3, 4$. We observe a couple of closed triangles in the quiver diagram in figure 6 that give rise to the following Yukawa couplings in the superpotential

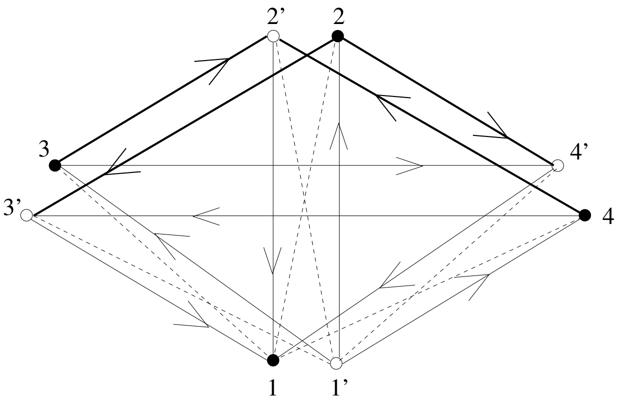

Figure 6: Quiver diagram for the branes $\{1, 2, 3, 4\}$.

$$\begin{aligned} &\Phi_{2'3}\, \Phi_{1'2}, h^{(2)}_{13}\,, \qquad &&\Phi_{2'3}\, \Phi_{1'3}, h^{(2)}_{12} \\ &\Phi_{2'4}\, \Phi_{1'2}, h^{(2)}_{14}\,, \qquad &&\Phi_{2'4}\, \Phi_{1'4}, h^{(2)}_{12}\,. \end{aligned} \tag{6.14}$$

Condensation of the chiral fields $\Phi_{2'3}$ and $\Phi_{2'4}$ leads to a mass matrix for the six fields $\{\Phi_{1'2}, \Phi_{1'3}, \Phi_{1'4}, h^{(2)}_{12}, h^{(2)}_{13}, h^{(2)}_{14}\}$ of rank four. Thus, one combination of the three fields Φ, one combination of the three fields $h^{(2)}$ and furthermore the three fields $h^{(1)}$ remain massless. These modes just fit into the three chiral fields in table 9 in addition to one further hypermultiplet in the $(4, 2, 1)$ representation of the Pati-Salam gauge group $U(4) \times$

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$U(2) \times U(2)$. The condensation for the second triplet of U(2)s is completely analogous and leads to a massless hypermultiplet in the $(4, 1, 2)$ representation.

The quiver diagram involving the six U(2) gauge groups is shown in figure 7. In this quiver diagram there are closed polygons like $(2 - 4' - 7' - 6)$ which after condensation generate a mass term for one chiral component inside each of the nine hypermultiplets $\{H_{25}, H_{26}, \ldots, H_{46}, H_{47}\}$. Remember that a hypermultiplet consists of two chiral multiplets of opposite charge, $H = (h^{(1)}, h^{(2)})$. The mass matrix for these nine chiral fields has rank six, so that three combinations of the four chiral fields, $h^{(1)}$, in $\{H_{36}, H_{37}, H_{46}, H_{47}\}$ remain massless. Since the intersection numbers in table 9 tell us that there are no chiral fields in the $(1, 2, 2)$ representation of the $U(4) \times U(2) \times U(2)$ gauge group, the other chiral components, $h^{(2)}$, of the hypermultiplets must also gain a mass during brane recombination. A very similar behavior was found in [21], and it was pointed out that this might involve the condensation of massive string modes, as well. These would at least allow the correct mass terms in the quiver diagram.

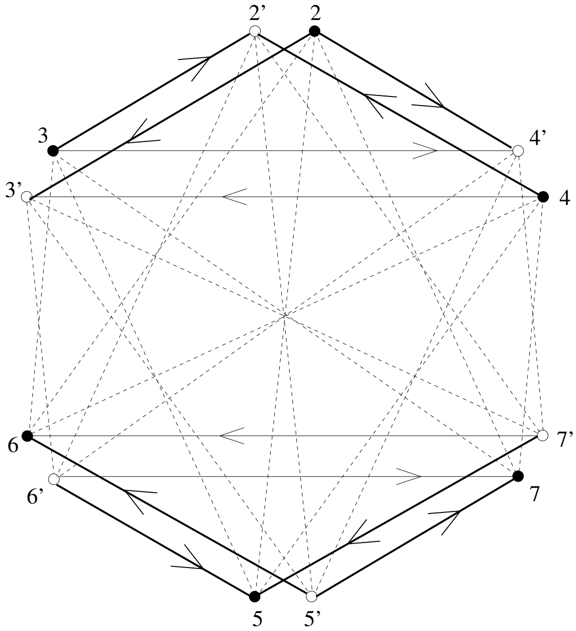

Figure 7: Quiver diagram for the branes $\{2, 3, 4, 5, 6, 7\}$.

We expect that the quiver diagram really tells us half of the complete story, so that the non-chiral spectrum of the three generation Pati-Salam model is as listed in table 10.

Intriguingly, these are just appropriate Higgs fields to break the Pati-Salam gauge group down to the Standard Model.

field	n	$U(4) \times U(2) \times U(2)$
H_{aa}	1	$(\mathrm{Adj}, 1, 1)$ + c.c.
H_{bb}	1	$(1, \mathrm{Adj}, 1)$ + c.c.
H_{cc}	1	$(1, 1, \mathrm{Adj})$ + c.c.
$H_{a'b}$	1	$(4, 2, 1)$ + c.c.
$H_{a'c}$	1	$(4, 1, 2)$ + c.c.
H_{bc}	3	$(1, 2, \overline{2})$ + c.c.

Table 10: Non-chiral spectrum for 3 stack PS-model.

6.4 Getting the Standard Model

It is beyond the scope of this paper to discuss all the phenomenological consequences of this 3 generation Pati-Salam model. However, we would like to present two possible ways of breaking the GUT Pati-Salam model down to the Standard Model.

6.4.1 Adjoint Pati-Salam breaking

There are still the adjoint scalars related to the unconstrained positions of the branes on the third T^2. By moving one of the four D6-branes away from the U(4) stack, or in other words by giving VEV to appropriate fields in the adjoint of U(4), we can break the gauge

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n	$\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)^4$	$\mathrm{U}(1)_{B-L}$
1	$(3,2,1)_{(1,1,0,0)}$	$\frac{1}{3}$
2	$(3,2,1)_{(1,-1,0,0)}$	$\frac{1}{3}$
1	$(\overline{3},1,2)_{(-1,0,-1,0)}$	$-\frac{1}{3}$
2	$(\overline{3},1,2)_{(-1,0,1,0)}$	$-\frac{1}{3}$
1	$(1,2,1)_{(0,1,0,1)}$	-1
2	$(1,2,1)_{(0,-1,0,1)}$	-1
1	$(1,1,2)_{(0,0,-1,-1)}$	1
2	$(1,1,2)_{(0,0,1,-1)}$	1
1	$(1,S+A,1)_{(0,2,0,0)}$	0
1	$(1,1,\overline{S}+\overline{A})_{(0,0,-2,0)}$	0

Table 11: Chiral spectrum for 4 stack left-right symmetric SM.

group down to $\mathrm{U}(3) \times \mathrm{U}(2) \times \mathrm{U}(2) \times \mathrm{U}(1)$. Indeed the resulting spectrum as shown in table 11 looks like a three generation left-right symmetric extension of the standard model.

Performing the anomaly analysis, one finds two anomaly free U(1)s, of which the combination $\frac{1}{3}(\mathrm{U}(1)_1 - 3\,\mathrm{U}(1)_4)$ remains massless even after the Green-Schwarz mechanism. This linear combination in fact is the $\mathrm{U}(1)_{B-L}$ symmetry, which is expected to be anomaly-free in a model with right-handed neutrinos.

By giving a VEV to fields in the adjoint of $\mathrm{U}(2)_R$, one obtains the next symmetry breaking, where the two $\mathrm{U}(2)_R$ branes split into two U(1) branes. This gives rise to the gauge symmetry $\mathrm{U}(3) \times \mathrm{U}(2)_L \times \mathrm{U}(1)_R \times \mathrm{U}(1)_R \times \mathrm{U}(1)$. In this case the following two U(1) gauge factors remain massless after checking the Green-Schwarz couplings

$$\begin{aligned} \mathrm{U}(1)_{B-L} &= \frac{1}{3}(\mathrm{U}(1)_1 - 3\,\mathrm{U}(1)_5) \\ \mathrm{U}(1)_Y &= \frac{1}{3}\,\mathrm{U}(1)_1 + \mathrm{U}(1)_3 - \mathrm{U}(1)_4 - \mathrm{U}(1)_5\,. \end{aligned} \tag{6.15}$$

It is very assuring that we indeed obtain a massless hypercharge. The final supersymmetric chiral spectrum is listed in table 12 with respect to the unbroken gauge symmetries.

The anomalous $\mathrm{U}(1)_1$ can be identified with the baryon number operator and survives the Green-Schwarz mechanism as a global symmetry. Therefore, in this model the baryon number is conserved and the proton is stable. Similarly, $\mathrm{U}(1)_5$ can be identified with the lepton number and also survives as a global symmetry. To break the gauge symmetry $\mathrm{U}(1)_{B-L}$, one can recombine the third and the fifth stack of D6 branes, which is expected to correspond to giving a VEV to the Higgs field $H_{3'5}$. We will see in section 6.4.2. that this brane recombination gives a mass to the right handed neutrino.

To proceed, let us compute the relation between the Standard Model gauge couplings at the PS-breaking scale at string tree level. The $\mathrm{U}(N_a)$ gauge couplings for D6-branes are given by

$$\frac{4\pi}{g_a^2} = \frac{M_s^3}{g_s}\,\mathrm{Vol}(\mathrm{D6}_a)\,, \tag{6.16}$$

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n	field	$\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)^3$	$\mathrm{U}(1)_Y \times \mathrm{U}(1)_{B-L}$
1	q_L	$(3,2)_{(1,1,0,0,0)}$	$\left(\frac{1}{3},\frac{1}{3}\right)$
2	q_L	$(3,2)_{(1,-1,0,0,0)}$	$\left(\frac{1}{3},\frac{1}{3}\right)$
1	u_R	$(\overline{3},1)_{(-1,0,-1,0,0)}$	$\left(-\frac{4}{3},-\frac{1}{3}\right)$
2	u_R	$(\overline{3},1)_{(-1,0,0,1,0)}$	$\left(-\frac{4}{3},-\frac{1}{3}\right)$
2	d_R	$(\overline{3},1)_{(-1,0,1,0,0)}$	$\left(\frac{2}{3},-\frac{1}{3}\right)$
1	d_R	$(\overline{3},1)_{(-1,0,0,-1,0)}$	$\left(\frac{2}{3},-\frac{1}{3}\right)$
1	l_L	$(1,2)_{(0,1,0,0,1)}$	$(-1,-1)$
2	l_L	$(1,2)_{(0,-1,0,0,1)}$	$(-1,-1)$
2	e_R	$(1,1)_{(0,0,1,0,-1)}$	$(2,1)$
1	e_R	$(1,1)_{(0,0,0,-1,-1)}$	$(2,1)$
1	ν_R	$(1,1)_{(0,0,-1,0,-1)}$	$(0,1)$
2	ν_R	$(1,1)_{(0,0,0,1,-1)}$	$(0,1)$
1		$(1,S+A)_{(0,2,0,0,0)}$	$(0,0)$
1		$(1,1)_{(0,0,-2,0,0)}$	$(-2,0)$
1		$(1,1)_{(0,0,0,-2,0)}$	$(2,0)$
2		$(1,1)_{(0,0,-1,-1,0)}$	$(0,0)$

Table 12: Chiral spectrum for 5 stack SM.

where $\mathrm{Vol}(D6_a)$ denotes the internal volume of the 3-cycle the D6-branes are wrapping on. During the brane recombination process the volume of the recombined brane is equal to the sum of the volumes of the two intersecting branes. Therefore, we have the following ratios for the volumes of the five stacks of D6-branes in our model

$$\mathrm{Vol}(\mathrm{D6}_2) = \mathrm{Vol}(\mathrm{D6}_3) = \mathrm{Vol}(\mathrm{D6}_4) = 3\,\mathrm{Vol}(\mathrm{D6}_1)\,, \qquad \mathrm{Vol}(\mathrm{D6}_5) = \mathrm{Vol}(\mathrm{D6}_1)\,. \tag{6.17}$$

This allows us at string tree level to determine the ratio of the Standard Model gauge couplings at the PS breaking scale to be

$$\frac{\alpha_s}{\alpha_Y} = \frac{11}{3}\,, \qquad \frac{\alpha_w}{\alpha_Y} = \frac{11}{9} \tag{6.18}$$

leading to a Weinberg angle $\sin^2\theta_w = 9/20$ which differs from the usual SU(5) GUT prediction $\sin^2(\theta_w) = 3/8$. Encouragingly, from (6.18) we get the right order for the sizes of the Standard Model gauge couplings constants, $\alpha_s > \alpha_w > \alpha_Y$. It would be interesting to analyze whether this GUT value is consistent with the low energy data at the weak scale. A potential problem is the appearance of colored Higgs fields in table 10, which would spoil the asymptotic freedom of the SU(3). In order to improve this situation one needs a model with less non-chiral matter, i.e. a model where not so many open string sectors actually preserve $\mathcal{N} = 2$ supersymmetry.

6.4.2 Bifundamental Pati-Salam breaking

We can also use directly the bifundamental Higgs fields like $H_{a'c}$ to break the model down to the Standard Model gauge group. This higgsing in string theory should correspond to a

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n	field	sector	$SU(3)_c \times SU(2)_L \times U(1)^4$	$U(1)_Y$
2	q_L	(AB)	$(3,2)_{(1,-1,0,0)}$	$\frac{1}{3}$
1	q_L	$(A'B)$	$(3,2)_{(1,1,0,0)}$	$\frac{1}{3}$
1	u_R	(AC)	$(\overline{3},1)_{(-1,0,1,0)}$	$-\frac{4}{3}$
2	d_R	$(A'C)$	$(\overline{3},1)_{(-1,0,-1,0)}$	$\frac{2}{3}$
2	u_R	(AD)	$(\overline{3},1)_{(-1,0,0,1)}$	$-\frac{4}{3}$
1	d_R	$(A'D)$	$(\overline{3},1)_{(-1,0,0,-1)}$	$\frac{2}{3}$
2	l_L	(BC)	$(1,2)_{(0,-1,1,0)}$	-1
1	l_L	$(B'C)$	$(1,2)_{(0,1,1,0)}$	-1
1	e_R	$(C'D)$	$(1,1)_{(0,0,-1,-1)}$	2
1	e_R	$(C'C)$	$(1,1)_{(0,0,-2,0)}$	2
1	e_R	$(D'D)$	$(1,1)_{(0,0,0,-2)}$	2
1	S	$(B'B)$	$(1,S+A)_{(0,2,0,0)}$	0

Table 13: Chiral spectrum for 4 stack SM.

recombination of one of the four D6-branes wrapping π_a with one of the branes wrapping π'_c. Thus, we get the following four stacks of D6-branes

$$\pi_A = \pi_a\,, \qquad \pi_B = \pi_b\,, \qquad \pi_C = \pi_a + \pi'_c\,, \qquad \pi_D = \pi_c \tag{6.19}$$

supporting the initial gauge group $U(3)\times U(2)\times U(1)^2$. The tadpole cancellation conditions are still satisfied. One gets the chiral spectrum by computing the homological intersection numbers as shown in table 13.

By computing the mixed anomalies, one finds that there are two anomalous $U(1)$ gauge factors and two anomaly free ones

$$\begin{aligned} U(1)_Y &= \frac{1}{3}\,U(1)_A - U(1)_C - U(1)_D \\ U(1)_K &= U(1)_A - 9\ U(1)_B + 9\ U(1)_C - 9\ U(1)_D\,. \end{aligned} \tag{6.20}$$

Remarkably, the axionic couplings just leave the hypercharge massless, so that we finally get the Standard Model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. In this model only the baryon number generator can be identified with $U(1)_1$, whereas the lepton number is broken. Therefore, the proton is stable and lepton number violating couplings as Majorana mass terms are possible. Note, that there are no massless right-handed neutrinos in this model. As we have mentioned already, this model is related to the model discussed in the last section by a further brane recombination process, affecting the mass of the right handed neutrinos. This brane recombination can be considered as a stringy mechanism to generate GUT scale masses for the right handed neutrinos [21]. The different ways of gauge symmetry breaking that have been discussed so far are depicted in figure 8.

It is evident from table 13 that there is also something unusually going on with the right handed leptons. Only one of them is realized as a bifundamental field, the remaining two are given by symmetric representations of $U(1)$. This behavior surely will have consequences for the allowed couplings, in particular for the Yukawa couplings and the electroweak Higgs mechanism.

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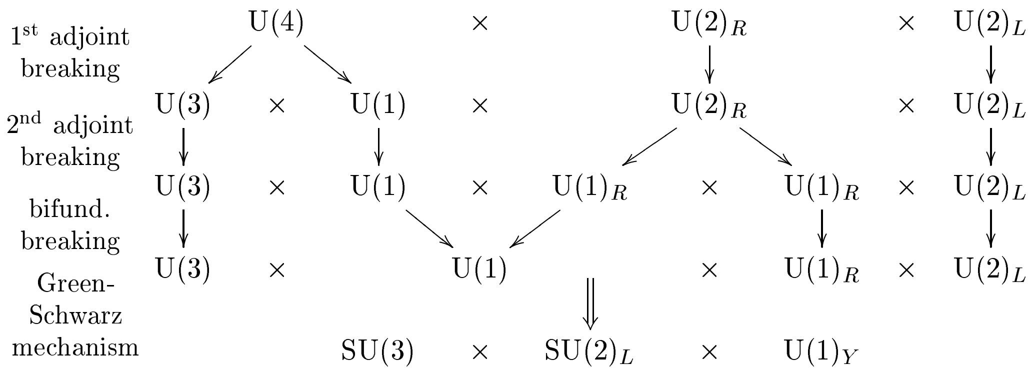

Figure 8: Gauge symmetry breaking of $\mathrm{U}(4) \times \mathrm{U}(2)_L \times \mathrm{U}(2)_R$.

Computing the gauge couplings, we find the following ratios for the internal volumes of the four 3-cycles

$$\mathrm{Vol}(\mathrm{D6}_2) = \mathrm{Vol}(\mathrm{D6}_4) = 3\,\mathrm{Vol}(\mathrm{D6}_1)\,, \qquad \mathrm{Vol}(\mathrm{D6}_3) = 4\,\mathrm{Vol}(\mathrm{D6}_1)\,. \tag{6.21}$$

This allows us to determine the ratio of the Standard Model gauge couplings at the GUT scale to be again

$$\frac{\alpha_s}{\alpha_Y} = \frac{11}{3}\,, \qquad \frac{\alpha_w}{\alpha_Y} = \frac{11}{9} \tag{6.22}$$

leading to a Weinberg angle $\sin^2\theta_w = 9/20$. Thus, both models provide the same prediction for the Weinberg-angle at the GUT scale.

6.4.3 Electroweak symmetry breaking

Finally, we would like to make some comments on electroweak symmetry breaking in this model. From the quiver diagram of the $\mathrm{U}(4) \times \mathrm{U}(2) \times \mathrm{U}(2)$ Pati-Salam model we do not expect that the three Higgs fields in the $(1, \overline{2}, 2)$ representation get a mass during the brane recombination process. Therefore, our model does contain appropriate Higgs fields to participate in the electroweak symmetry breaking. The three Higgs fields, H_{bc}, in the Pati-Salam model in table 10 give rise to the Higgs fields

$$H_{BD} = (1,2)_{(0,1,0,-1)} + \mathrm{c.c.}\,, \qquad H_{B'C} = (1,2)_{(0,1,1,0)} + \mathrm{c.c.} \tag{6.23}$$

for the $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ model above.

Of course supersymmetry should already be broken by some mechanism above the electroweak symmetry breaking scale, but nevertheless we can safely discuss the expectations from the purely topological data of the corresponding brane recombination process. Since we do not want to break the color SU(3), we still take a stack of three D6-branes which are wrapped on the cycle $\pi_\alpha = \pi_A$. Giving a VEV to the fields H_{BD} is expected to correspond to the brane recombination

$$\pi_\beta = \pi_B + \pi_D\,. \tag{6.24}$$

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However, for the brane recombination

$$\pi_\gamma = \pi_B + \pi'_C \,, \tag{6.25}$$

the identification with the corresponding field theory deformation is slightly more subtle, as the intersections between these two branes support both the massless chiral multiplet $l_L^{B'C}$ as listed in table 13 and the Higgs field $H_{B'C}$. Thus, the intersection preserves only $\mathcal{N}=1$ supersymmetry and one might expect that some combination of $l_L^{B'C}$ and $H_{B'C}$ are involved in the brane recombination process. Even without knowing all the details, in the following we can safely compute the chiral spectrum via intersection numbers.

After the brane recombination we have a naive gauge group $\mathrm{U}(3)\times\mathrm{U}(1)\times\mathrm{U}(1)$, which however is broken by the Green-Schwarz couplings to $\mathrm{SU}(3)_c\times\mathrm{U}(1)_{em}$ with

$$\mathrm{U}(1)_{em} = \frac{1}{6}\,\mathrm{U}(1)_\alpha - \frac{1}{2}\,\mathrm{U}(1)_\beta + \frac{1}{2}\,\mathrm{U}(1)_\gamma \,. \tag{6.26}$$

Interestingly, just $\mathrm{U}(1)_{em}$ survives this brane recombination process. Moreover, all intersection numbers vanish, so that there are no chiral massless fields, i.e. all quark and leptons in table 13 have gained a mass including the left-handed neutrinos and the exotic matter. Looking at the charges in table 13, one realizes that in the leptonic sector this Higgs effect cannot be the usual one, where simply l_L and e_R receive a mass via some Yukawa couplings. Here also higher dimensional couplings, like the dimension five coupling

$$W \sim \frac{1}{M_s}\overline{H}_{BD}\,\overline{H}_{BD}\,S\,e_R^{D'D} \,, \tag{6.27}$$

are relevant. These couplings induce a mixing of the Standard Model matter with the exotic field, S. Thus we can state, that by realizing some of the right handed leptons in the (anti-)symmetric representation, the exotic field is needed to give all leptons a mass during electroweak symmetry breaking. It remains to be seen whether the induced masses can be consistent with the low-energy data.

7. Conclusions

In this paper we have studied intersecting brane worlds for the $T^6/\mathbb{Z}_4$ orientifold background with special emphasis on supersymmetric configurations. We have found as a first non-trivial result a globally supersymmetric three generation Pati-Salam type extension of the Standard Model with some exotic matter. The chiral matter content is only slightly extended by one chiral multiplet in the (anti-)symmetric representation of $\mathrm{SU}(2)_L$. The presence of this exotic matter can be traced back to the fact that we were starting with a Pati-Salam gauge group, where the anomaly constraints forced us to introduce additional matter. Issues which arose for non-supersymmetric models will also appear in the supersymmetric setting. Since the Green-Schwarz mechanism produces global U(1) symmetries, the allowed couplings in the effective gauge theory are usually much more constrained than for the Standard Model.

With such model a hand, many phenomenological issues deserve to be studied, as for instance mechanisms for supersymmetry breaking, the generation of soft breaking terms, Yukawa and higher dimensional couplings, the generation of μ-terms and gauge coupling unification. It also remains to be seen whether the electroweak Higgs effect indeed produces the correct masses for all quarks and leptons. Moreover, one should check whether the renormalization of the gauge couplings from the string respectively the PS-breaking scale down to the weak scale can lead to acceptable values for the Weinberg angle.

The motivation for this analysis was to start a systematic search for realistic supersymmetric intersecting brane world models. We have worked out some of the technical model building aspects when one is dealing with more complicated orbifold backgrounds containing in particular twisted sector 3-cycles. These techniques can be directly generalized to, for instance, the $\mathbb{Z}_6$ orientifolds [48] or the $\mathbb{Z}_N \times \mathbb{Z}_M$ orientifold models [51]. It could be worthwhile to undertake a similar study for these orbifold models, too.

The final goal would be to find a realization of the MSSM in some simple intersecting brane world model. As should have become clear from our analysis, while phenomenologically interesting non-supersymmetric models are fairly easy to get, the same is not true for the supersymmetric ones. Requiring supersymmetry imposes very strong constraints on the possible configurations and as we have observed in the $\mathbb{Z}_4$ example, also the supply of possible intersection numbers is very limited. These obstructions appear to be less surprising, when one contemplates that for smooth backgrounds, by lifting to M-theory, the construction of an $\mathcal{N} = 1$ chiral intersecting brane world background with $O6$ planes and D6 branes is equivalent to the construction of a compact singular G_2 manifold. In this respect it would be interesting whether certain M-theory orbifold constructions like the one discussed in [67] are dual to the kind of models discussed in this paper.

At a certain scale close to the TeV scale supersymmetry has to be broken. For the intersecting brane world scenario one might envision different mechanisms for such a breaking. First, we might use the conventional mechanism of gaugino condensation via some non-perturbative. Alternatively, one could build models where the MSSM is localized on a number of D-branes, but where the RR-tadpole cancellation conditions requires the introduction of hidden sector branes, on which supersymmetry might be broken. This breaking could be mediated gravitationally to the standard model branes. A third possibility is to get D-term supersymmetry breaking by generating effective Fayet-Iliopoulos terms via complex structure deformations. We think that these issues and other phenomenological questions deserve to be studied in the future.

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A. Orientifold planes

In this appendix we present the results for the O6-planes and the action of $\Omega\overline{\sigma}$ on the homology lattice for the other three orientifold models. We have listed the results in table 14.

model	O6-plane
AAA	$4\,\rho_1 - 2\,\overline{\rho}_2$
AAB	$2\,\rho_1 + \rho_2 - 2\,\overline{\rho}_2$
ABA	$2\,\rho_1 + 2\,\rho_2 + 2\,\overline{\rho}_1 - 2\,\overline{\rho}_2$
ABB	$2\,\rho_2 + 2\,\overline{\rho}_1 - 2\,\overline{\rho}_2$

Table 14: O6-planes.

For the action of $\Omega\overline{\sigma}$ on the orbifold basis we find:

AAA: For the toroidal 3-cycles we get

$$\begin{aligned} \rho_1 &\to \rho_1\,, & \overline{\rho}_1 &\to -\overline{\rho}_1\,, \\ \rho_2 &\to -\rho_2\,, & \overline{\rho}_2 &\to \overline{\rho}_2 \end{aligned} \tag{A.1}$$

and for the exceptional cycles

$$\varepsilon_i \to \varepsilon_i \qquad \overline{\varepsilon}_i \to -\overline{\varepsilon}_i\,, \tag{A.2}$$

for all $i \in \{1, \ldots, 6\}$.

AAB: For the toroidal 3-cycles we get

$$\begin{aligned} \rho_1 &\to \rho_1\,, & \overline{\rho}_1 &\to \rho_1 - \overline{\rho}_1\,, \\ \rho_2 &\to -\rho_2\,, & \overline{\rho}_2 &\to -\rho_2 + \overline{\rho}_2 \end{aligned} \tag{A.3}$$

and for the exceptional cycles

$$\varepsilon_i \to \varepsilon_i \qquad \overline{\varepsilon}_i \to \varepsilon_i - \overline{\varepsilon}_i\,, \tag{A.4}$$

for all $i \in \{1, \ldots, 6\}$.

ABA: For the toroidal 3-cycles we get

$$\begin{aligned} \rho_1 &\to \rho_2\,, & \overline{\rho}_1 &\to -\overline{\rho}_2 \\ \rho_2 &\to \rho_1\,, & \overline{\rho}_2 &\to -\overline{\rho}_1 \end{aligned} \tag{A.5}$$

and for the exceptional cycles

$$\begin{aligned} \varepsilon_1 &\to -\varepsilon_1 & \overline{\varepsilon}_1 &\to \overline{\varepsilon}_1 \\ \varepsilon_2 &\to -\varepsilon_2 & \overline{\varepsilon}_2 &\to \overline{\varepsilon}_2 \\ \varepsilon_3 &\to \varepsilon_3 & \overline{\varepsilon}_3 &\to -\overline{\varepsilon}_3 \\ \varepsilon_4 &\to \varepsilon_4 & \overline{\varepsilon}_4 &\to -\overline{\varepsilon}_4 \\ \varepsilon_5 &\to \varepsilon_6 & \overline{\varepsilon}_5 &\to -\overline{\varepsilon}_6 \\ \varepsilon_6 &\to \varepsilon_5 & \overline{\varepsilon}_6 &\to -\overline{\varepsilon}_5\,. \end{aligned} \tag{A.6}$$

B. Supersymmetry conditions

In this appendix we list the supersymmetry conditions for the remaining three orientifold models.

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AAA: The condition that such a D6-brane preserves the same supersymmetry as the orientifold plane is simply

$$\varphi_{a,1} + \varphi_{a,2} + \varphi_{a,3} = 0 \bmod 2\pi \tag{B.1}$$

with

$$\tan\varphi_{a,1} = \frac{m_{a,1}}{n_{a,1}}, \qquad \tan\varphi_{a,2} = \frac{m_{a,2}}{n_{a,2}}, \qquad \tan\varphi_{a,3} = \frac{U_2\, m_{a,3}}{n_{a,3}}. \tag{B.2}$$

This implies the following necessary condition in terms of the wrapping numbers

$$U_2 = -\frac{n_{a,3}}{m_{a,3}} \frac{(n_{a,1}\, m_{a,2} + m_{a,1}\, n_{a,2})}{(n_{a,1}\, n_{a,2} - m_{a,1}\, m_{a,2})}. \tag{B.3}$$

AAB: The condition that such a D6-brane preserves the same supersymmetry as the orientifold plane is simply

$$\varphi_{a,1} + \varphi_{a,2} + \varphi_{a,3} = 0 \bmod 2\pi \tag{B.4}$$

with

$$\tan\varphi_{a,1} = \frac{m_{a,1}}{n_{a,1}}, \qquad \tan\varphi_{a,2} = \frac{m_{a,2}}{n_{a,2}}, \qquad \tan\varphi_{a,3} = \frac{U_2\, m_{a,3}}{n_{a,3} + \frac{1}{2} m_{a,3}}. \tag{B.5}$$

This implies the following necessary condition in terms of the wrapping numbers

$$U_2 = -\frac{\left(n_{a,3} + \frac{1}{2} m_{a,3}\right)}{m_{a,3}} \frac{(n_{a,1}\, m_{a,2} + m_{a,1}\, n_{a,2})}{(n_{a,1}\, n_{a,2} - m_{a,1}\, m_{a,2})}. \tag{B.6}$$

ABA: The condition that such a D6-brane preserves the same supersymmetry as the orientifold plane is simply

$$\varphi_{a,1} + \varphi_{a,2} + \varphi_{a,3} = \frac{\pi}{4} \bmod 2\pi \tag{B.7}$$

with

$$\tan\varphi_{a,1} = \frac{m_{a,1}}{n_{a,1}}, \qquad \tan\varphi_{a,2} = \frac{m_{a,2}}{n_{a,2}}, \qquad \tan\varphi_{a,3} = \frac{U_2\, m_{a,3}}{n_{a,3}}. \tag{B.8}$$

This implies the following necessary condition in terms of the wrapping numbers

$$U_2 = \frac{n_{a,3}}{m_{a,3}} \frac{(n_{a,1}\, n_{a,2} - m_{a,1}\, m_{a,2} - n_{a,1}\, m_{a,2} - m_{a,1}\, n_{a,2})}{(n_{a,1}\, n_{a,2} - m_{a,1}\, m_{a,2} + n_{a,1}\, m_{a,2} + m_{a,1}\, n_{a,2})}. \tag{B.9}$$

C. Fractional boundary states

The unnormalized boundary states in light cone gauge for D6-branes at angles in the untwisted sector are given by

$$\begin{aligned} |D;(n_I,m_I)\rangle_U = {} & |D;(n_I,m_I),NSNS,\eta=1\rangle_U + |D;(n_I,m_I),NSNS,\eta=-1\rangle_U + \\ & + |D;(n_I,m_I),RR,\eta=1\rangle_U + |D;(n_I,m_I),RR,\eta=-1\rangle_U \end{aligned} \tag{C.1}$$

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with the coherent state

$$
\begin{aligned}
|D;(n_I,m_I),\eta\rangle = \int dk_2 dk_3 \sum_{\vec{r},\vec{s}} \exp\Big(& -\sum_{\mu=2}^{3}\sum_{n>0}\frac{1}{n}\alpha^{\mu}_{-n}\tilde{\alpha}^{\mu}_{-n} - \\
& -\sum_{I=1}^{3}\sum_{n>0}\frac{1}{2n}\left(e^{2i\varphi_I}\zeta^I_{-n}\tilde{\zeta}^I_{-n}+e^{-2i\varphi_I}\overline{\zeta}^I_{-n}\tilde{\overline{\zeta}}^I_{-n}\right)+ \\
& + i\eta[\text{fermions}]\Big)|\vec{r},\vec{s},\vec{k},\eta\rangle\,. \qquad \text{(C.2)}
\end{aligned}
$$

Here α^μ denotes the two real non-compact directions and ζ^I the three complex compact directions. The angles φ_I of the D6-brane relative to the horizontal axis on each of the three internal tori T^2 can be expressed by the wrapping numbers (n_I, m_I) as listed in appendix B. The boundary state (C.2) involves a sum over the internal Kaluza-Klein and winding ground states parameterized by $(\vec{r},\vec{s})$. The mass of these KK and winding modes on each T^2 in general reads

$$
M_I^2 = \frac{|r_I + s_I\,U_I|^2}{U_{I,2}}\,\frac{|n_I + m_I\,T_I|^2}{T_{I,2}} \qquad \text{(C.3)}
$$

with $r_I, s_I \in \mathbb{Z}$ as above and U_I and T_I denote the complex and Kähler structure on the torus [45]. If the brane carries some discrete Wilson lines, $\vartheta = 1/2$, appropriate factors of the form $e^{isR\vartheta}$ have to be introduced into the winding sum in (C.2).

In the Θ^2 twisted sector, the boundary state involves the analogous sum over the fermionic spin structures (C.1) with

$$
\begin{aligned}
|D;(n_I,m_I),e_{ij},\eta\rangle_T = \int dk_2 dk_3 \sum_{r_3,s_3} \exp\Big(& -\sum_{\mu=2}^{3}\sum_{n>0}\frac{1}{n}\alpha^{\mu}_{-n}\tilde{\alpha}^{\mu}_{-n} - \\
& -\sum_{I=1}^{2}\sum_{r\in\mathbb{Z}_0^+ + \frac{1}{2}}\frac{1}{2r}\left(e^{2i\varphi_I}\zeta^I_{-r}\tilde{\zeta}^I_{-r}+e^{-2i\varphi_I}\overline{\zeta}^I_{-r}\tilde{\overline{\zeta}}^I_{-r}\right)- \\
& -\sum_{n>0}\frac{1}{2n}\left(e^{2i\varphi_3}\zeta^3_{-n}\tilde{\zeta}^3_{-n}+e^{-2i\varphi_3}\overline{\zeta}^3_{-n}\tilde{\overline{\zeta}}^3_{-n}\right)+ \\
& + i\eta[\text{fermions}]\Big)|r_3,s_3,\vec{k},e_{ij},\eta\rangle\,. \qquad \text{(C.4)}
\end{aligned}
$$

where e_{ij} denote the 16 $\mathbb{Z}_2$ fixed points. Here, we have taken into account that the twisted boundary state can only have KK and winding modes on the third T^2 torus and that the bosonic modes on the two other T^2 tori carry half-integer modes.

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